ECED 3300
Electromagnetic Fields
Final Examination

Place: Sexton Gymnasium

Instructor: Sergey A. Ponomarenko

Date and Time: December 14, 2009, 12:00 to 15:00 pm

Closed Books: Formula sheets are provided; absolutely no supplemental material and no calculators are allowed.

Problem 1 (20pts)

The space in between two concentric spheres of radii a and b, $(a < b)$, is filled with a conducting material with the conductivity $\sigma(r) = k/r$, where k is a given constant. Determine the resistance between the spheres.

Problem 2 (15pts)

Given the magnetic field distribution in free space:

$$\mathbf{H}(\rho, \phi) = \begin{cases} H_0 \left(\frac{\rho^2}{R^2}\right) \cos 2\phi \, \mathbf{a}_z, & \rho \leq R, \\ H_0 \left(\frac{R^2}{\rho^2}\right) \cos 2\phi \, \mathbf{a}_z, & \rho \geq R, \end{cases}$$

where H_0 and R are constants, determine the current density distribution everywhere.

Problem 3 (20pts)

Two homogeneous magnetic regions $z \leq 0$ (region 1) and $z \geq 0$ (region 2) have permeabilities $\mu_1 = 5\mu_0$ and $\mu_2 = 10\mu_0$, respectively. The surface current distribution at the interface between the media is given, $\mathbf{K} = 3\mathbf{a}_x$, A/m. Given the magnetic field in region 1, $\mathbf{H}_1 = 5\mathbf{a}_y + 10\mathbf{a}_z$ A/m, find the magnetic field in region 2.

Problem 4 (20pts)

Determine the work done to assemble in free space a charged sphere of radius R with a uniform volume charge density ρ_0.

Problem 5 (25pts)

A uniformly charged line of length $2l$ with the charge density ρ_l is placed along the $x-$axis. Find the electrostatic potential at **any** point P in the **plane** bisecting the charged line, as shown in the figure below. Note: the problem is three dimensional, and the figure gives a cross-sectional view.

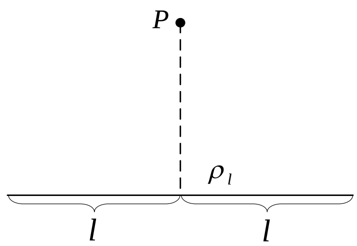

FIG. 1: Illustration to Problem 5

Bonus Problem (40pts)

An insulated coil with current is wound on the surface of a sphere of radius a in such a way as to produce a uniform magnetic field H_0 in the $z-$direction inside the sphere and a dipole field outside, i.e,

$$\mathbf{H}(r,\theta) = \begin{cases} H_0\mathbf{a}_z, & r \leq a, \\ \frac{H_0 a^3}{2r^3}(2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta), & r \geq a. \end{cases}$$

The medium inside and outside the sphere has a uniform permeability μ. Determine

(a) the necessary surface current density $\mathbf{K}$;

(b) a vector potential – both inside and outside the sphere – giving rise to the field distribution.

This is a more challenging problem and it would seem prudent to tackle it last.