ECED 3300
Electromagnetic Fields
Final Examination

Place: Sexton Gymnasium

Instructor: Sergey A. Ponomarenko

Date and Time: Monday, December 11, 2017, 12 to 15 pm.

Closed Books: Formula sheets are provided; absolutely no supplemental material and no calculators are allowed.

Hint: Make sure to justify all your answers to get full credit.

Problem 1 (15pts)

Given a time-dependent electric field, $\mathbf{E} = A(y\mathbf{a}_x - x\mathbf{a}_y)\cos\omega t$, V/m, where A is a known constant, find the time rate of change of the **magnetic flux density** everywhere in space.

Problem 2 (20pts)

Two concentric spheres of radii a and b, $(b > a)$ are filled with a dielectric material such that the dielectric permittivity of the system is given by

$$\epsilon = \begin{cases} \epsilon_1, & r < a; \\ \epsilon_2, & a < r < b. \end{cases}$$

The spheres are situated in **free space**, and a point charge Q is placed at the center of the spheres as is indicated in the figure.

a) Determine the electric field **everywhere**;

b) Find the amount of electrostatic energy stored in the **shaded area**;

c) Find the volume polarization charge density **everywhere**.

d) Determine the amount of work required to move a point charge q to any point on the surface of the outer sphere, $r = b$, from far away.

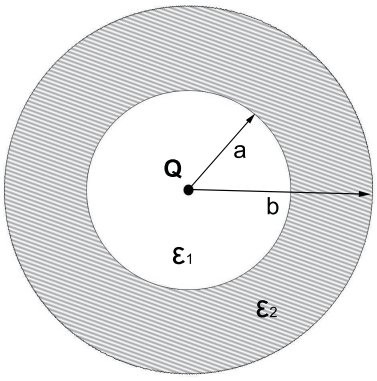

Fig. 1

FIG. 1: Illustration to Problem 2.

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Problem 3 (20pts)

Given the vector field $\mathbf{F}$ such that

$$\mathbf{F} = \frac{e^{-\rho^2}}{\rho}\cos\phi\,\mathbf{a}_\rho + 2\rho\, e^{-\rho^2}\sin\phi\,\mathbf{a}_\phi.$$

a) Determine whether $\mathbf{F}$ can represent a **magnetic flux density**.

b) Find the flux of $\mathbf{F}$ through a cylinder of height H and radius R, centered at the origin.

Problem 4 (15pts)

Given the magnetic field,

$$\mathbf{H}(\mathbf{r}) = \frac{2\cos\theta}{r}\mathbf{a}_r - \frac{\sin\theta}{r}\mathbf{a}_\theta, \quad \mathrm{A/m},$$

a) determine the current density generating this magnetic field;

b) find the time rate of change of the charge density at any point in space;

c) determine the total current through a sphere of radius R centered at the origin.

Problem 5 (30pts)

The upper half-space $z > 0$ is a dielectric medium with the permittivity ϵ. The lower half-space $z < 0$ is empty and is free of charge. The volume charge density in the **dielectric** is given by the expression

$$\rho_v(z) = \rho_0 e^{-z/a},$$

where ρ_0 and a are known constants.

(a) Determine the electrostatic potential in the **dielectric**.

(b) Find the electric field **everywhere**.

Hint: You may express your answer in terms of ϵ_0.