ECED 3300
Electromagnetic Fields
Final Examination

Place: Sexton Gymnasium

Instructor: Sergey A. Ponomarenko

Date and Time: Saturday, December 15, 2018, 8:30 to 11:30 am.

Closed Books: Formula sheets are provided; absolutely no supplemental material and no calculators are allowed.

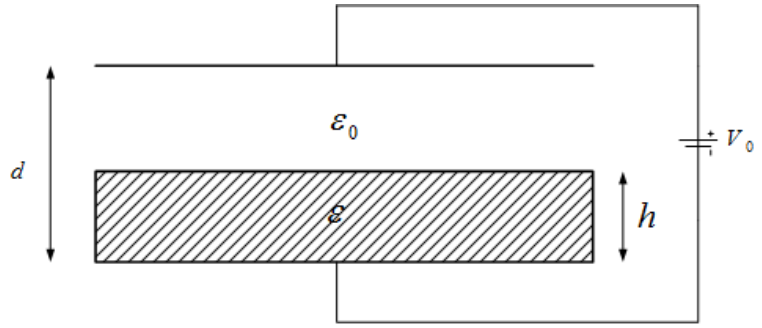
Hint: Make sure to justify all your answers to get full credit.

Problem 1 (15pts)

A parallel plate capacitor, displayed in the figure below, is partially filled with a dielectric of permittivity ϵ and is connected to a dc voltage V_0. The area of the capacitor plates is A.

(a) What is the capacitance of such a capacitor?

(b) Determine the energy stored in the capacitor.



Problem 2 (15pts)

A conducting loop depicted in Fig.2 is a sector of a circle of radius R, subtending the angle α with the x-axis in the xy-plane. The loop is placed in an inhomogeneous time-varying magnetic field $\mathbf{B}(x,t) = \mathbf{a}_z B_0 x \cos\omega t$. Find the electromotive force induced in the loop.

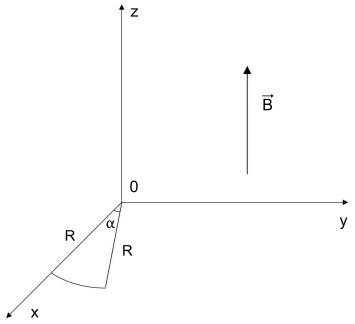


Fig. 2

Problem 3 (20pts)

Given the vector potential distribution

$$\mathbf{A}(\mathbf{r}) = \begin{cases} \frac{B_0 r \sin\theta}{2}\,\mathbf{a}_\phi, & r \le a; \\ \frac{B_0 a^3 \sin\theta}{2r^2}\,\mathbf{a}_\phi, & r \ge a, \end{cases}$$

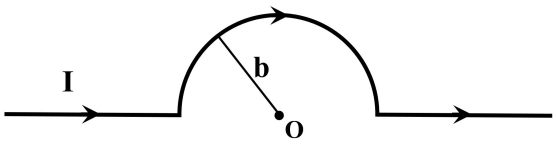
where B_0 and a are known constants,

(a) determine the magnetic flux density **everywhere**.

(b) How much energy is stored in a sphere of radius a, centered at the origin?

Problem 4 (20pts)

A filamentary wire, carrying the current I, consists of two straight segments—see the figure below—and a semicircular loop of radius b, centered at the point O. The current direction is indicated by arrows. Find the magnetic field at the point O.



Problem 5 (30pts)

The upper $z > 0$ (lower, $z < 0$) half-space is filled with a dielectric medium with the permittivity $\epsilon_>$ ($\epsilon_<$). There exists a volume charge density

$$\rho_v(\mathbf{r}) = \rho_0 e^{-\kappa|z|},$$

where ρ_0 and κ are known constants.

(a) What is the electrostatic potential everywhere?

(b) Determine the electric field everywhere;

(c) Find the surface charge density on the plane $z = 0$.