Midterm Examination, ECED 3300

Instructor: Sergey A. Ponomarenko.
Place, Date & Time: Sexton Gym; November 8, 2018, 8:35 to 10:35 am
Closed Books: Formula sheets are provided; no calculators are allowed.
Hint: Make sure to justify all your answers to get full credit.

Problem 1 (15pts)

This problem consists of two unrelated parts.

a) Determine the volume charge density generating the following electric flux density field

$$\mathbf{D}(\rho, \phi, z) = \left(\frac{e^{-\rho^2}}{\rho}\right) \cos\phi\, \mathbf{a}_\rho + \rho e^{-\rho^2} \sin\phi\, \mathbf{a}_\phi + e^{-\rho^2} \mathbf{a}_z, \qquad \text{C/m}^2.$$

b) What condition must any true **electrostatic** field satisfy?

Problem 2 (25pts)

Consider a charged spherical cloud of radius a with a uniform charge density ρ_v, assembled in free space.

a) Determine the electric field inside and outside the cloud.

b) Determine the electrostatic potential at the center of the cloud.

c) How much work does an external agent have to do to move a point charge q from far away to the center of the cloud.

Problem 3 (25pts)

A spherical capacitor consists of two concentric spheres of radii R_1 and R_2, ($R_2 > R_1$). The inner and outer spheres carry charges $+Q$ and $-Q$, respectively. The space in between the spheres is filled with a dielectric with the permittivity

$$\epsilon = \begin{cases} \epsilon_1, & R_1 \le r \le a; \\ \epsilon_2, & a \le r \le R_2. \end{cases}$$

Determine the electrostatic energy stored in the capacitor.

Problem 4 (35pts)

Point charges Q_1 and Q_2 are located at the points $(-a, 0, h)$ and $(a, 0, h)$ in the **Cartesian** coordinates **above** an infinite, grounded conducting plane $z = 0$. Answer the following questions:

(a) **Justify** your choice of the magnitude(s), sign(s), and locations(s) of all image charge(s);

(b) Find the total force (in the vector form) experienced by the charge Q_1.

(c) Determine the work done to remove the charge Q_2 far away from the plane.

Hint: You might find useful the following trigonometric identities

$$\cos\alpha = \frac{1}{\sqrt{1+\tan^2\alpha}}; \qquad \sin\alpha = \frac{\tan\alpha}{\sqrt{1+\tan^2\alpha}}.$$