ECED 4310
Tutorial: Plane electromagnetic waves

Problem 1

A plane electromagnetic wave propagating in a nonmagnetic medium is specified by its magnetic field,

$$\mathbf{H} = 25\sin(2\times 10^8 t + 6x)\mathbf{a}_y,\ mA/m.$$

Determine:

1. The direction of wave propagation.
2. The permittivity of the medium.
3. The electric field intensity.

Solution

1. The phase can be written as $\theta = \omega t + \beta x$, where $\omega = 2\times 10^8$ s^{-1} and $\beta = 6$. Thus, the wave propagates in a negative x-direction (backward-propagating wave).
2. $$\beta = \omega\sqrt{\epsilon\mu_0} = \frac{\omega}{c}\sqrt{\epsilon_r}.$$
   It follows that
   $$\epsilon_r = \frac{\beta^2 c^2}{\omega^2} = 81, \Longrightarrow \underline{\epsilon = 81\epsilon_0}.$$
3. $$\mathbf{H} = \mathrm{Re}[\mathbf{H}_0 e^{j(2\times 10^8 t + 6x)}],$$
   where
   $$\mathbf{H}_0 = -25j\mathbf{a}_y,\ mA/m. \tag{1}$$
   It follows that
   $$\mathbf{E}_0 = -\eta(\mathbf{a}_k\times\mathbf{H}_0), \qquad \mathbf{a}_k = -\mathbf{a}_x, \qquad \eta = \sqrt{\mu_0/\epsilon} = \eta_0/9 = 40\pi/3. \tag{2}$$

It can be inferred from Eqs. (1) and (2) that

$$\mathbf{E}_0 = \frac{\eta_0}{9}(\mathbf{a}_x \times \mathbf{H}_0) = -\frac{25\eta_0}{9} j\mathbf{a}_z,\ mV/m$$

Thus,

$$\mathbf{E} = \text{Re}[\mathbf{E}_0 e^{j(2\times 10^8 t + 6x)}] = \underline{\frac{1}{3}\mathbf{a}_z \sin(2\times 10^8 t + 6x)},\ V/m.$$

Problem 2

The electric field of a plane wave propagating in a nonpermeable dielectric medium is given by

$$\mathbf{E} = \mathbf{a}_x 2\cos(10^8 t - z/\sqrt{3}) - \mathbf{a}_y \sin(10^8 t - z/\sqrt{3}),\ V/m.$$

1. Determine the frequency and wavelength of the wave.
2. What is the dielectric constant of the medium?
3. Describe the wave polarization.
4. Find the corresponding $\mathbf{H}$.

Solution

1. The phase of the wave is $\theta = \omega t - \beta z$. Thus, $\underline{\omega = 10^8}\ \text{s}^{-1}$; $\underline{\lambda = 2\pi/\beta = 2\pi\sqrt{3}}$.
2. $\beta = \omega\sqrt{\mu\epsilon} = \frac{\omega}{c}\sqrt{\epsilon_r}$. It follows that $\underline{\epsilon_r = \beta^2 c^2/\omega^2 = 3}$.
3. Since $|E_{0x}| \neq |E_{0y}|$ and $\phi_{0x} \neq \phi_{0y}$, the wave is (left-hand) elliptically polarized.
4. The complex amplitude of the electric field is

$$\mathbf{E}_0 = 2\mathbf{a}_x + j\mathbf{a}_y,$$

Also, $\eta = \eta_0/\sqrt{\epsilon_r} = 120\pi/\sqrt{3}$ and $\mathbf{a}_k = \mathbf{a}_z$. Thus,

$$\mathbf{H}_0 = \frac{\mathbf{a}_k \times \mathbf{E}_0}{\eta} = \frac{\sqrt{3}}{120\pi}(2\mathbf{a}_y - j\mathbf{a}_x).$$

Hence,

$$\underline{\mathbf{H} = \text{Re}[\mathbf{H}_0 e^{j(\beta z - \omega t)}] = \frac{\sqrt{3}}{120\pi}\left[\mathbf{a}_y 2\cos(10^8 t - z/\sqrt{3}) + \mathbf{a}_x \sin(10^8 t - z/\sqrt{3})\right]},\ A/m.$$

Problem 3

The magnetic field at the entrance to a good conductor, filling the half-space $y \geq 0$, is given by the expression

$$\mathbf{H}(0,t) = h_0(\mathbf{a}_x \cos\omega t - \mathbf{a}_z \sin\omega t),$$

where h_0 is a known real constant. Determine $\mathbf{E}(y,t)$ and $\mathbf{H}(y,t)$ inside the medium.

Solution

Recall that for a good conductor,

$$\gamma = \frac{1+j}{\delta}, \qquad \eta = \frac{1+j}{\sigma\delta}, \qquad \theta_\eta = \pi/4.$$

The complex amplitude of the magnetic field at the entrance to the medium is then

$$\mathbf{H}_0 = \text{Re}[h_0(\mathbf{a}_x + j\mathbf{a}_y)]. \tag{3}$$

Thus, the magnetic field at any $y \geq 0$ is just an inhomogeneous plane wave,

$$\mathbf{H}(y,t) = \text{Re}\left[h_0(\mathbf{a}_x + j\mathbf{a}_y)e^{-y/\delta}e^{j(y/\delta-\omega t)}\right].$$

It then follows that

$$\mathbf{E}_0 = -\eta(\mathbf{a}_k \times \mathbf{H}_0), \qquad \mathbf{a}_k = \mathbf{a}_y. \tag{4}$$

It then follows from Eqs. (3) and (4) that

$$\mathbf{E}_0 = -\frac{(1+j)}{\sigma\delta}[\mathbf{a}_y \times (\mathbf{a}_x + j\mathbf{a}_z)] = -\frac{h_0}{\sigma\delta}(1+j)(-\mathbf{a}_z + j\mathbf{a}_x) = \frac{h_0\sqrt{2}}{\sigma\delta}e^{j\pi/4}(\mathbf{a}_z - j\mathbf{a}_x). \tag{5}$$

It can be inferred from Eqs. (3) and (5) that

$$\mathbf{E}_0 \cdot \mathbf{H}_0 = 0,$$

as expected for a plane wave. Finally,

$$\underline{\mathbf{H}(y,t) = h_0 e^{-y/\delta}[\mathbf{a}_x \cos(y/\delta - \omega t) - \mathbf{a}_z \sin(y/\delta - \omega t)]}.$$

and

$$\underline{\mathbf{E}(y,t) = \frac{h_0\sqrt{2}}{\sigma\delta}e^{-y/\delta}[\mathbf{a}_z \cos(y/\delta - \omega t + \pi/4) + \mathbf{a}_x \sin(y/\delta - \omega t + \pi/4)]}.$$

Notice that

$$\mathbf{E} \cdot \mathbf{H} \neq 0,$$

i.e., the orthogonality between the electric and magnetic fields is lost thanks to the phase lag of 45° between the fields.