ECED 4310

Tutorial: Reflection and Refraction of Electromagnetic Waves

Problem: Normal Reflection by a Good Conductor.

A plane electromagnetic wave in air with $\mathbf{E}_i = \mathbf{a}_x E_0 e^{j(\beta_0 z - \omega t)}$ is incident normally onto the surface at $z = 0$ of a highly conducting medium with the constitutive parameters σ, ϵ, and μ such that $\sigma/\epsilon\omega \gg 1$.

- Find the power reflection coefficient.
- Determine the fraction of the wave power absorbed by the medium.

Solution

- The impedances of the two media are $\eta_1 = \sqrt{\mu_0/\epsilon_0} = \eta_0$ in air and

  $$\eta_2 = \eta_c = \sqrt{\frac{\mu/\epsilon}{1 - \frac{j\sigma}{\epsilon\omega}}} \simeq \sqrt{\frac{j\mu\omega}{\sigma}},$$

  in the conductor. The reflection coefficient is given by the expression,

  $$r = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_c - \eta_0}{\eta_c + \eta_0}.$$

  As $|\eta_c| \ll \eta_0$, we can expand it into a Taylor series and keep terms only to the lowest order in $|\eta_c|/\eta_0$ to obtain

  $$r \simeq \left[\left(1 - \frac{\eta_c}{\eta_0}\right)\left(1 - \frac{\eta_c}{\eta_0}\right)\right] = 1 - \frac{2\eta_c}{\eta_0}.$$

  Next,

  $$\eta_c = e^{j\pi/4}\sqrt{\frac{\mu\omega}{\sigma}} = \frac{1+j}{\sigma\delta},$$

  in terms of the skin depth δ. Hence, to the first order in η_c/η_0, we obtain,

  $$R = |r|^2 = \left|1 - \frac{2}{\sigma\delta\eta_0} - \frac{2j}{\sigma\delta\eta_0}\right|^2 \simeq 1 - \frac{4}{\sigma\delta\eta_0}.$$

- The incident wave can be written as

  $$\mathbf{E}_i = \mathbf{a}_x E_0 e^{j(\beta_0 z - \omega t)}, \qquad \mathbf{H}_i = \mathbf{a}_y \frac{E_0}{\eta_0} e^{j(\beta_0 z - \omega t)}.$$

The reflected wave is specified by

$$\mathbf{E}_r = \mathbf{a}_x E_r e^{-j(\beta_0 z - \omega t)}, \qquad \mathbf{H}_r = -\mathbf{a}_y \frac{E_r}{\eta_0} e^{-j(\beta_0 z - \omega t)}.$$

Also, $E_r = rE_0$; the Poynting vectors for the incident and reflected waves are

$$\langle \boldsymbol{\mathcal{P}}_i \rangle = \mathbf{a}_z \frac{|E_0|^2}{2\eta_0}, \qquad \langle \boldsymbol{\mathcal{P}}_r \rangle = -\mathbf{a}_z \frac{R|E_0|^2}{2\eta_0}.$$

By the energy conservation, the absorbed power is given by,

$$|\langle \boldsymbol{\mathcal{P}}_a \rangle| = |\langle \boldsymbol{\mathcal{P}}_i \rangle| - |\langle \boldsymbol{\mathcal{P}}_r \rangle|.$$

which makes the fraction F,

$$F = \frac{|\langle \boldsymbol{\mathcal{P}}_a \rangle|}{|\langle \boldsymbol{\mathcal{P}}_i \rangle|} = 1 - R = \frac{4}{\sigma \delta \eta_0}.$$

of the incident power.

Problem: Retro-reflection.

Determine the fraction of the incident power reflected back by the retro-reflecting setup shown in the figure. The isosceles triangular prizm is made of glass with $\epsilon_r = 4$ and the light is incident from air.

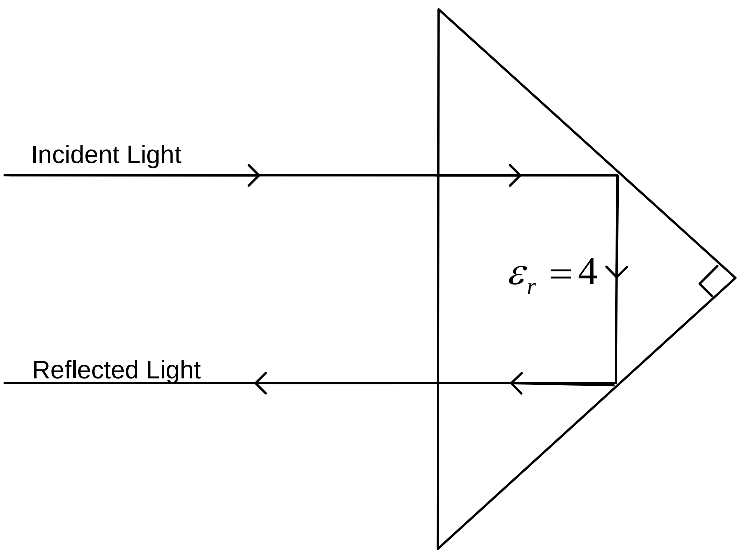

FIG. 1: Illustration to Problem 2.

Solution

In this problem $\eta_1 = \eta_0$ and $\eta_2 = \eta_0/\sqrt{\epsilon_r} = \eta_0/2$. The air-glass and glass-air transmission coefficients are

$$t_{12} = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2}{1+\sqrt{\epsilon_r}} = 2/3.$$

and

$$t_{21} = \frac{2\eta_1}{\eta_1 + \eta_2} = \frac{2\sqrt{\epsilon_r}}{1+\sqrt{\epsilon_r}} = 4/3.$$

As the angle of incidence within the glass is $\pi/4$, which is greater than the critical angle for total internal reflection, $\theta_c = \sin^{-1}(1/\sqrt{\epsilon_r}) = \sin^{-1}(1/2) = \pi/6$, the wave stays within the glass and is reflected back into the air. The transmitted field amplitude in the glass is $E_{t1} = t_{12}E_i$, and the transmitted back field amplitude is $E_{t2} = t_{21}E_{t1} = t_{12}t_{21}E_i$. The incident and transmitted back powers are $P_i = |E_i|^2/2\eta_0$ and $P_t = |E_{t2}|^2/2\eta_0$, respectively, implying that

$$F = \frac{P_t}{P_i} = \frac{|E_{t2}|^2}{|E_i|^2} = |t_{12}t_{21}|^2 = \frac{64}{81},$$

which is about 80%.