Midterm Examination, ECED 3300

Instructor: Sergey A. Ponomarenko.

Place, Date and Time: Sexton Campus; Fri, November 4 2011, 11:35-1:35 pm.

Closed Books: Formula sheets are provided; no calculators are allowed.

Hint: Make sure to justify all your answers to get full credit.

Problem 1 (20pts)

There is a vector field distribution $\mathbf{E} = \rho\cos\phi\mathbf{a}_\rho + C\rho\sin\phi\,\mathbf{a}_\phi + \mathbf{a}_z$ V/m in free space, where C is a constant. Determine:

(a) the conditions under which $\mathbf{E}$ is a genuine electrostatic field;

(b) the charge distribution that can generate $\mathbf{E}$ in free space.

You may leave your answers in terms of ϵ_0.

Problem 2 (15pts)

If the potential distribution is $V(x, y, z) = xyz$, calculate the electrostatic energy in the region of free space specified by $0 \le x \le 1$, $0 \le y \le 1$, and $0 \le z \le 1$. You may leave your answers in terms of ϵ_0.

Problem 3 (30pts)

A spherical cavity of radius a, centered at the origin, is filled with a dielectric of permittivity $\epsilon_<$. The cavity is surrounded by a dielectric medium with the permittivity $\epsilon_>$. The electrostatic potential is specified by

$$V(r,\theta) = \begin{cases} (V_0 r/a)\cos\theta, & r \le a, \\ (V_0 a^2/r^2)\cos\theta, & r \ge a, \end{cases}$$

where V_0 is a known constant. Find

(a) the electric field everywhere;

(b) the volume density of polarized charges everywhere;

(c) the surface charge density on the sphere.

Problem 4 (35pts)

Suppose you are given a time-dependent current density

$$\mathbf{J}(r,t) = J_0 \frac{r^2}{R^2}\, e^{-r/R}\, e^{-t/\tau}\mathbf{a}_r, \quad \text{A/m}^2,$$

where J_0 and τ are known constants. Determine

(a) the total current through a sphere of radius R, centered at the origin at any time t;

(b) the volume charge density at any point inside the sphere at the time t, given a uniform initial charge density ρ_{v0} there.