(c) $A\times B=\begin{vmatrix}4 & -6 & 3\\ -1 & 8 & 5\end{vmatrix}=(-30-24)a_x+(-3-20)a_y+(32-6)a_z$

$=\underline{\underline{-54a_x-23a_y+26a_z}}$

Prob. 1.5

$B\times C=\begin{vmatrix}3 & 5 & 1\\ 0 & 1 & -7\end{vmatrix}=(-35-1)a_x+(0+21)a_y+(3-0)a_z$

$=-36a_x+21a_y+3a_z$

$A\bullet(B\times C)=(4,2,1)\bullet(-36,21,3)=-144+42+3=\underline{\underline{-99}}$

Prob. 1.6

(a) $B\times C=\begin{vmatrix}1 & 1 & 1\\ 0 & 1 & 2\end{vmatrix}=a_x-2a_y+a_z$

$A\bullet(B\times C)=(1,0,-1)\bullet(1,-2,1)=1+0-1=\underline{\underline{0}}$

(b) $A\times B=\begin{vmatrix}1 & 0 & -1\\ 1 & 1 & 1\end{vmatrix}=a_x-2a_y+a_z$

$(A\times B)\bullet C=(1,-2,1)\bullet(0,1,2)=0-2+2=\underline{\underline{0}}$

(c) $A\times(B\times C)=\begin{vmatrix}1 & 0 & -1\\ 1 & -2 & 1\end{vmatrix}=\underline{\underline{-2a_x-2a_y-2a_z}}$

(d) $(A\times B)\times C=\begin{vmatrix}1 & -2 & 1\\ 0 & 1 & 2\end{vmatrix}=\underline{\underline{-5a_x-2a_y+a_z}}$

Prob.1.7

(a) T = (3, -2, 1) and S = (4, 6, 2)

(b) $r_{TS}=r_s-r_t=(4, 6, 2)-(3, -2, 1)=\underline{a_x+8a_y+a_z}$

(c) distance = $|r_{TS}|=\sqrt{1+64+1}=\underline{\underline{8.124}}$ m

Prob. 1.8

(a) If **A** and **B** are parallel, **A**=k**B**, where k is a constant.

$$(\alpha,3,-2)=k(4,\beta,8)$$

Equating coefficients gives

$$-2 = 8k \longrightarrow k = -\frac{1}{4}$$
$$\alpha = 4k = \underline{\underline{-1}}$$
$$3 = \beta k \longrightarrow \beta = 3/k = \underline{\underline{-12}}$$

This can also be solved using **A X B** = 0.

(b) If A and B are perpendicular to each other,

$$\mathbf{A} \bullet \mathbf{B} = 0 \longrightarrow \underline{\underline{4\alpha + 3\beta - 16 = 0}}$$

Prob. 1.9

(a) $A \cdot B = \text{AB} \cos \theta_{AB}$

$A \times B = \text{AB} \sin \theta_{AB} a_n$

$$(A \cdot B)^2 + |A \times B|^2 = (AB)^2 (\cos^2 \theta_{AB} + \sin^2 \theta_{AB}) = (\text{AB})^2$$

(b) $\boldsymbol{a}_x \cdot (\boldsymbol{a}_y \times \boldsymbol{a}_z) = \boldsymbol{a}_x \cdot \boldsymbol{a}_x = 1$. Hence,

$$\frac{\boldsymbol{a}_y \times \boldsymbol{a}_z}{\boldsymbol{a}_x \cdot \boldsymbol{a}_y \times \boldsymbol{a}_z} = \frac{\boldsymbol{a}_x}{1} = \boldsymbol{a}_x$$

$$\frac{\boldsymbol{a}_z \times \boldsymbol{a}_x}{\boldsymbol{a}_x \cdot \boldsymbol{a}_y \times \boldsymbol{a}_z} = \frac{\boldsymbol{a}_y}{1} = \boldsymbol{a}_y$$

$$\frac{\boldsymbol{a}_x \times \boldsymbol{a}_y}{\boldsymbol{a}_x \cdot \boldsymbol{a}_y \times \boldsymbol{a}_z} = \frac{\boldsymbol{a}_z}{1} = \boldsymbol{a}_z$$

Prob. 1.10

(a) $\boldsymbol{P} + \boldsymbol{Q} = (6,2,0), \boldsymbol{P} + \boldsymbol{Q} - \boldsymbol{R} = (7,1,-2)$

$$|\boldsymbol{P} + \boldsymbol{Q} - \boldsymbol{R}| = \sqrt{49+1+4} = \sqrt{54} = \underline{\underline{7.3485}}$$

(b) $$\boldsymbol{P}.\boldsymbol{Q} \times \boldsymbol{R} = \begin{vmatrix} 2 & -1 & -2 \\ 4 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} = 2(6-2) + (8+2) - 2(4+3) = 8 + 10 - 14 = \underline{\underline{4}}$$

$$\boldsymbol{Q} \times \boldsymbol{R} = \begin{vmatrix} 4 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} = (4,-10,7)$$

$$\boldsymbol{P}.\boldsymbol{Q} \times \boldsymbol{R} = (2,-1,-2) \cdot (4,-10,7) = 8 + 10 - 14 = \underline{\underline{4}}$$

$$OR = \sqrt{49+100+16} = \sqrt{165} = 12.845$$

$$t = \frac{OR}{v} = \frac{12.845}{300} = \underline{\underline{42.82 \text{ ms}}}$$

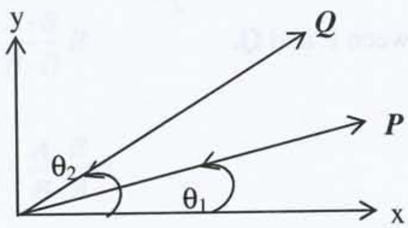
Prob. 1.19

$$\text{Area} = \frac{1}{2}|\boldsymbol{D}\times\boldsymbol{E}| = \frac{1}{2}\begin{vmatrix} 4 & 1 & -5 \\ -1 & 2 & 3 \end{vmatrix} = \frac{1}{2}|(3+10)\boldsymbol{a}_x + (5-12)\boldsymbol{a}_y + (8+1)\boldsymbol{a}_z|$$

$$= \frac{1}{2}|(13,-7,9)| = \frac{1}{2}\sqrt{169+49+81} = \underline{\underline{8.646}}$$

Prob. 1.20

(a) Let **P** and **Q** be as shown below:



$$|\boldsymbol{P}| = \cos^2\theta_1 + \sin^2\theta_1 = 1, |\boldsymbol{Q}| = \cos^2\theta_2 + \sin^2\theta_2 = 1,$$

Hence **P** and **Q** are unit vectors.

(b) $\boldsymbol{P}\cdot\boldsymbol{Q} = (1)(1)\cos(\theta_2\text{-}\theta_1)$

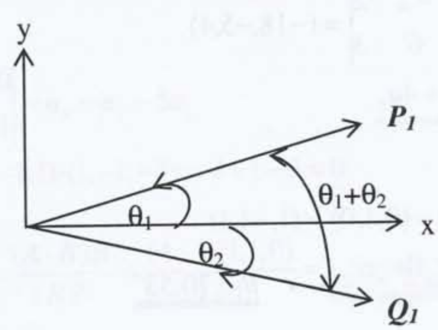
But $\boldsymbol{P}\cdot\boldsymbol{Q} = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$. Thus,

$$\underline{\underline{\cos(\theta_2-\theta_1) = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2}}$$

Let $\boldsymbol{P}_1 = \boldsymbol{P} = \cos\theta_1\boldsymbol{a}_x + \sin\theta_1\boldsymbol{a}_y$ and

$$\boldsymbol{Q}_1 = \cos\theta_2\boldsymbol{a}_x - \sin\theta_2\boldsymbol{a}_y.$$

$\boldsymbol{P_1}$ and $\boldsymbol{Q_1}$ are unit vectors as shown below:



$$P_1 \cdot Q_1 = (1)(1)\cos(\theta_1 + \theta_2)$$

But $P_1 \cdot Q_1 = \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2$,

$$\cos(\theta_2 + \theta_1) = \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2$$

Alternatively, we can obtain this formula from the previous one by replacing θ_2 by $-\theta_2$ in $\boldsymbol{Q}$.

(c)

$$\frac{1}{2}|P-Q| = \frac{1}{2}|(\cos\theta_1 - \cos\theta_2)a_x + (\sin\theta_1 - \sin\theta_2)a_y$$

$$= \frac{1}{2}\sqrt{\cos^2\theta_1 + \sin^2\theta_1 + \cos^2\theta_2 + \sin^2\theta_2 - 2\cos\theta_1\cos\theta_2 - 2\sin\theta_1\sin\theta_2}$$

$$= \frac{1}{2}\sqrt{2 - 2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2)} = \frac{1}{2}\sqrt{2 - 2\cos(\theta_2 - \theta_1)}$$

Let $\theta_2 - \theta_1 = \theta$, the angle between **P** and **Q.**

$$\frac{1}{2}|P-Q| = \frac{1}{2}\sqrt{2-2\cos\theta}$$

But $\cos 2A = 1 - 2\sin^2 A$.

$$\frac{1}{2}|P-Q| = \frac{1}{2}\sqrt{2-2+4\sin^2\theta/2} = \sin\theta/2$$

Thus,

$$\frac{1}{2}|P-Q| = |\sin\frac{\theta_2-\theta_1}{2}|$$

Prob. 1.21

$$w = \frac{w(1,-2,2)}{3} = (1,-2,2), \ r = r_p - r_o = (1,3,4) - (2,-3,1) = (-1,6,3)$$

$$u = w \times r = \begin{vmatrix} 1 & -2 & 2 \\ -1 & 6 & 3 \end{vmatrix} = (-18,-5,4)$$

$$u = -18a_x - 5a_y + 4a_z$$

Prob. 1.22

$r_1 = (1,1,1)$, $\quad r_2 = (1,0,1) - (0,1,0) = (1,-1,1)$

$$\cos\theta = \frac{r_1 \cdot r_2}{r_1 r_2} = \frac{(1-1+1)}{\sqrt{3}\sqrt{3}} = \frac{1}{3} \quad \longrightarrow \quad \theta = 70.53^o$$

Prob. 1.23

(a) $T_s = T \cdot a_s = \frac{T \cdot S}{|S|} = \frac{(2,-6,3)\cdot(1,2,1)}{\sqrt{6}} = \frac{-7}{\sqrt{6}} = \underline{\underline{-2.8577}}$

(b) $\boldsymbol{S_T} = (\boldsymbol{S} \cdot \boldsymbol{a_T})\boldsymbol{a_T} = \frac{(\boldsymbol{S} \cdot \boldsymbol{T})\boldsymbol{T}}{\boldsymbol{T}^2} = \frac{-7(2,-6,3)}{7^2}$

$= \underline{\underline{-0.2857\boldsymbol{a}_x + 0.8571\boldsymbol{a}_y - 0.4286\boldsymbol{a}_z}}$

(c) $\sin\theta_{TS} = \frac{|\boldsymbol{T} \times \boldsymbol{S}|}{|\boldsymbol{T}||\boldsymbol{S}|} = \begin{vmatrix} 2 & -6 & 3 \\ 1 & 2 & 1 \end{vmatrix} = \frac{|(-12,1,10)|}{7\sqrt{6}} = \frac{\sqrt{245}}{7\sqrt{6}} = 0.9129$

$\Rightarrow \theta_{TS} = \underline{\underline{65.91^\circ}}$

Prob. 1.24

Let $\boldsymbol{A} = \boldsymbol{A}_{B\|} + \boldsymbol{A}_{B\perp}$

$\boldsymbol{A}_{B\|} = (\boldsymbol{A} \cdot \boldsymbol{a}_B)\boldsymbol{a}_B = \frac{\boldsymbol{A} \cdot \boldsymbol{B}}{\boldsymbol{B} \cdot \boldsymbol{B}}\boldsymbol{B}$

Hence,

$\boldsymbol{A}_{B\perp} = \boldsymbol{A} - \boldsymbol{A}_{B\|} = \boldsymbol{A} - \frac{\boldsymbol{A} \cdot \boldsymbol{B}}{\boldsymbol{B} \cdot \boldsymbol{B}}\boldsymbol{B}$

Prob.1.25

(a) $\boldsymbol{H}(1,3,-2) = 6\boldsymbol{a}_x + \boldsymbol{a}_y + 4\boldsymbol{a}_z$

$\boldsymbol{a}_H = \frac{(6,1,4)}{\sqrt{36+1+16}} = \underline{\underline{0.8242\boldsymbol{a}_x + 0.1374\boldsymbol{a}_y + 0.5494\boldsymbol{a}_z}}$

(b) $|\boldsymbol{H}| = 10 = \sqrt{4x^2y^2 + (x+z)^2 + z^4}$

or

$\underline{\underline{100 = 4x^2y^2 + x^2 + 2xz + z^2 + z^4}}$

Prob. 1.26

$C = 5\boldsymbol{a}_x + \boldsymbol{a}_z$

(a) $B \times C = \begin{vmatrix} 1 & 1 & 0 \\ 5 & 0 & 1 \end{vmatrix} = \boldsymbol{a}_x - \boldsymbol{a}_y - 5\boldsymbol{a}_z$

$A \bullet (B \times C) = (4,-1,1) \bullet (1,-1,-5) = 4 + 1 - 5 = \underline{\underline{0}}$

(b) $A_B = (A \bullet \boldsymbol{a}_B)\boldsymbol{a}_B = \frac{(\boldsymbol{A} \cdot \boldsymbol{B})\boldsymbol{B}}{|\boldsymbol{B}|^2} = \frac{(4-1)(1,1,0)}{1+1} = \underline{\underline{1.5\boldsymbol{a}_x + 1.5\boldsymbol{a}_y}}$

P.E. 3.10

$$\nabla\times\nabla V = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial V}{\partial x} & \frac{\partial V}{\partial y} & \frac{\partial V}{\partial z} \end{vmatrix} =$$

$$= \left(\frac{\partial^2 V}{\partial y\partial z} - \frac{\partial^2 V}{\partial y\partial z}\right)\boldsymbol{a}_x + \left(\frac{\partial^2 V}{\partial x\partial z} - \frac{\partial^2 V}{\partial z\partial x}\right)\boldsymbol{a}_y + \left(\frac{\partial^2 V}{\partial x\partial y} - \frac{\partial^2 V}{\partial y\partial x}\right)\boldsymbol{a}_z = 0$$

P.E. 3.11

(a)

$$\nabla^2 U = \frac{\partial}{\partial x}(2xy + yz) + \frac{\partial}{\partial y}(x^2 + xz) + \frac{\partial}{\partial z}(xy)$$

$$= \underline{\underline{2y.}}$$

(b)

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho(z\sin\phi + 2\rho) + \frac{1}{\rho^2}(-\rho z\sin\phi - 2z^2\frac{\partial}{\partial\rho}\sin\phi\cos\phi) + \frac{\partial}{\partial z}(\rho\sin\phi + 2z\cos^2\phi)$$

$$= \frac{1}{\rho}(z\sin\phi + 4\rho) - \frac{1}{\rho^2}(z\rho\sin\phi + 2z^2\cos 2\phi) + 2\cos^2\phi.$$

$$= \underline{\underline{4 + 2\cos^2\phi - \frac{2z^2}{\rho^2}\cos 2\phi.}}$$

(c)

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2\frac{1}{r}\cos\theta\sin\phi + 2r^3\phi] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}[-\sin^2\theta\sin\phi\ln r]$$

$$+ \frac{1}{r^2\sin^2\theta}[-\cos\theta\sin\phi\ln r]$$

$$= \underline{\underline{\frac{1}{r^2}\cos\theta\sin\phi(1 - 2\ln r - \csc^2\theta\ln r) + 6\phi}}$$

Prob. 3.11

$$\psi = \int_S \boldsymbol{A} \bullet d\boldsymbol{S} = \int\int z dx dz = \int_0^1 dx \int_0^2 z dz = (1)\frac{z^2}{2}\Big|_0^2 = \underline{\underline{2}}$$

Prob. 3.12

$$\psi = \int_S \boldsymbol{B} \bullet d\boldsymbol{S}, \quad d\boldsymbol{S} = dx dy \boldsymbol{a}_z$$

$$\psi = \int_{y=0}^{2} \int_{x=0}^{1} 3x^2 y dx dy = 3\frac{x^3}{3}\Big|_0^1 \frac{y^2}{2}\Big|_0^2 = (1)(2) = \underline{\underline{2}}$$

Prob. 3.13

(a) dv = dxdydz

$$\int_v xy dv = \int_{z=0}^{2} \int_{y=0}^{1} \int_{z=0}^{1} xy dx dy dz = \int_0^1 x dx \int_0^1 y dy \int_0^z dz$$

$$= \frac{x^2}{2}\Big|_0^1 \frac{y^2}{2}\Big|_0^1 z\Big|_0^2 = (1/2)(1/2)(2) = \underline{\underline{0.5}}$$

(b)

$$dv = \rho d\rho d\phi dz$$

$$\int_v \rho z dv = \int_{\phi=0}^{\pi} \int_{z=0}^{2} \int_{\rho=1}^{3} \rho z \, \rho d\rho d\phi dz = \int_1^3 \rho^2 d\rho \int_0^2 z dz \int_0^{\pi} d\phi$$

$$= \frac{\rho^3}{3}\Big|_1^3 \frac{z^2}{2}\Big|_0^2 (\pi) = (9 - \frac{1}{3})(2\pi) = \underline{\underline{54.45}}$$

Prob. 3.14

Let $\quad I = \int_v \boldsymbol{A} dv = \int_v r \sin\phi \boldsymbol{a}_r dv$

$\boldsymbol{a}_r = \sin\theta\cos\phi \boldsymbol{a}_x + \sin\theta\sin\phi \boldsymbol{a}_y + \cos\theta \boldsymbol{a}_z$

$A_r = r\sin\theta\sin\phi\cos\phi \boldsymbol{a}_x + r\sin\theta\sin^2\phi \boldsymbol{a}_y + r\cos\theta\sin\phi \boldsymbol{a}_z$

$dv = r^2 \sin\theta d\theta d\phi dr$

Prob. 3.16

(a)
$$\nabla U = \frac{\partial U}{\partial x}\boldsymbol{a}_x + \frac{\partial U}{\partial y}\boldsymbol{a}_y + \frac{\partial U}{\partial z}\boldsymbol{a}_z$$
$$= \underline{\underline{e^{x+2y}\cosh z\boldsymbol{a}_x + 2e^{x+2y}\cosh z\boldsymbol{a}_y + e^{x+2y}\sinh z\boldsymbol{a}_z}}$$

(b)
$$\nabla T = \frac{\partial T}{\partial \rho}\boldsymbol{a}_\rho + \frac{1}{\rho}\frac{\partial T}{\partial \phi}\boldsymbol{a}_\phi + \frac{\partial T}{\partial z}\boldsymbol{a}_z$$
$$= \underline{\underline{-\frac{3z}{\rho^2}\cos\phi\boldsymbol{a}_\rho - \frac{3z}{\rho^2}\sin\phi\boldsymbol{a}_\phi + \frac{3}{\rho}\cos\phi\boldsymbol{a}_z}}$$

$$\nabla W = \frac{\partial W}{\partial r}\boldsymbol{a}_r + \frac{1}{r}\frac{\partial W}{\partial \theta}\boldsymbol{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial W}{\partial \phi}\boldsymbol{a}_\phi$$

(c)
$$= \underline{\underline{\left(-\frac{5\cos\theta}{r^2} + 4r\sin\phi\right)\boldsymbol{a}_r + \left[2r^2\cos\phi - \frac{5\sin\phi}{r}\right]\frac{1}{r\sin\theta}\boldsymbol{a}_\phi}}$$

Prob. 3.17

$$r = \sqrt{x^2+y^2+z^2}, \qquad r^n = (x^2+y^2+z^2)^{n/2}$$

Method 1:

$$\nabla r^n = \frac{\partial r^n}{\partial x}\boldsymbol{a}_x + \frac{\partial r^n}{\partial y}\boldsymbol{a}_y + \frac{\partial r^n}{\partial z}\boldsymbol{a}_z = \frac{n}{2}(x^2+y^2+z^2)^{n/2-1}(2x)\boldsymbol{a}_x + \cdots$$
$$= n(x^2+y^2+z^2)^{n-2/2}(x\boldsymbol{a}_x + y\boldsymbol{a}_y + z\boldsymbol{a}_z) = \underline{\underline{nr^{n-2}\boldsymbol{r}}}$$

Method 2:

$$\nabla r^n = \frac{\partial r^n}{\partial r}\boldsymbol{a}_r = nr^{n-1}\frac{\boldsymbol{r}}{r} = nr^{n-2}\boldsymbol{r}$$

Prob. 3.18

$$\nabla T = 2x\boldsymbol{a}_x + 2y\boldsymbol{a}_y - \boldsymbol{a}_z$$

At $(1,1,2), \nabla T = (2,2,-1)$. The mosquito should move in the direction of

$$\underline{\underline{2\boldsymbol{a}_x + 2\boldsymbol{a}_y - \boldsymbol{a}_z}}$$

Prob. 3.30

$$\int_S \boldsymbol{H} \bullet d\boldsymbol{S} = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} 10\cos\theta r^2 \sin\theta d\theta d\phi \bigg|_{r=1}$$

$$= 10(1)^2 \int_0^{2\pi} d\phi \int_0^{\pi/2} \sin\theta\cos\theta d\theta = 10(2\pi)\int_0^{\pi/2} \sin\theta d(\sin\theta)$$

$$= 20\pi\left(\frac{\sin^2\theta}{2}\right)\bigg|_0^{\pi/2} = 10\pi = \underline{\underline{31.416}}$$

Prob. 3.31

$$\oint_S \boldsymbol{H}\cdot d\boldsymbol{S} = \int_v \nabla\cdot\boldsymbol{H} dv$$

$$\oint_S \boldsymbol{H}\cdot d\boldsymbol{S} = -\iint 2xydydz\bigg|_{x=0} + \iint 2xydydz\bigg|_{x=1} - \iint (x^2+z^2)dxdz\bigg|_{y=1}$$

$$+\iint (x^2+z^2)dxdz\bigg|_{y=2} - \iint 2yzdxdy\bigg|_{z=-1} + \iint 2yzdxdy\bigg|_{z=3}$$

$$= 0 + 2\int_1^2 ydy\int_{-1}^3 dz + 2\int_0^1 dx\int_1^2 ydy + 6\int_0^1 dx\int_1^2 ydy$$

$$= 12 + 3 + 9 = \underline{\underline{24}}$$

$$\nabla\cdot\boldsymbol{H} = \frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 2y + 0 + 2y = 4y$$

$$\int_V \nabla\cdot\boldsymbol{H} dv = \iiint 4ydxdydz = 4\int_0^1 dx\int_1^2 ydy\int_{-1}^3 dz$$

$$= 4(1)\frac{y^2}{2}\bigg|_1^2 (3+1) = \underline{\underline{24}}$$

Since A has no $\phi-$component, the first two integrals on the right hand side vanish

$$\int A \bullet dS = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{\pi/2} r^4 \sin\theta \, d\theta \, d\phi \Big|_{r=3} + \int_{r=0}^{3} \int_{\phi=0}^{\pi/2} r^2 \sin^2\theta \cos\phi \, dr \, d\phi \Big|_{\theta=\pi/2}$$

$$= 81 \left(\frac{\pi}{2}\right) (-\cos\theta)\Big|_0^{\pi/2} + 9(1)\sin\phi\Big|_0^{\pi/2}$$

$$= \frac{81\pi}{2} + 9 \quad = \underline{\underline{136.23}}$$

Prob. 3.35

Let $\psi = \oint F \bullet dS = \psi_t + \psi_b + \psi_o + \psi_i$

where $\psi_t, \psi_b, \psi_o, \psi_i$ are the fluxes through the top surface, bottom surface, outer surface ($\rho = 3$), and inner surface respectively.

For the top surface, $dS = \rho \, d\phi \, d\rho \, a_z$, $\quad z = 5$;

$F \bullet dS = \rho^2 \; z \; d\phi \; dz$. Hence:

$$\psi_t = \int_{\rho=2}^{3} \int_{\phi=0}^{2\pi} \rho^2 \; z \; d\phi \; dz \Big|_{z=5} = \frac{190\ \pi}{3} = 198.97$$

For the bottom surface, $\quad z = 0$, $dS = \rho \, d\phi \, d\rho(-a_z)$

$F \bullet dS = -\rho^2 z \, d\phi \; d\rho = 0$. Hence, $\psi_b = 0$.

For the outer curved surface, $\rho = 3$, $\quad dS = \rho d\phi dz \, \mathbf{a}_\rho$

$F \bullet dS = \rho^2 \sin\phi \; d\phi \; dz$. Hence,

$$\psi_a = \int_{z=0}^{5} dz \; \rho^3 \int_{\phi=0}^{2\pi} \sin\phi \, d\phi \Big|_{\rho=3} = 0$$

For the inner curved surface, $\rho = 2$, $\quad dS = \rho \, d\phi \, dz(-a_\rho)$

$F \bullet dS = -\rho^3 \sin\phi \; d\phi \; dz$. Hence,

$$\psi_a = -\int_{z=0}^{5} dz \; \rho^3 \int_{\phi=0}^{2\pi} \sin\phi \, d\phi \Big|_{\rho=2} = 0$$

$$\psi = \frac{190\pi}{3} + 0 + 0 + 0 = \frac{190\ \pi}{3} = \underline{\underline{198.97}}$$

$$\psi = \oint F \bullet dS = \int \nabla \bullet F dV$$

$$\nabla \bullet F = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho^3 \sin\phi) + \frac{1}{\rho}\frac{\partial}{\partial \phi}(z\cos\phi) + \rho$$

$$= 3\rho\sin\phi - \frac{z}{\rho}\sin\phi + \rho$$

$$\int_v \nabla \bullet F dv = \iiint (3\rho\sin\phi - \frac{z}{\rho}\sin\phi + \rho)\ \rho d\phi d\rho dz$$

$$= 0 + 0 + \int_0^5 dz \int_0^{2\pi} d\phi \int_2^3 \rho^2 d\rho$$

$$= \frac{190\pi}{3} = 198.97$$

Prob. 3.36

(a) $$\nabla x A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & y^2 & -xz \end{vmatrix} = z a_y - x a_z$$

$$\nabla x B = \left(\frac{1}{\rho} 2\rho z 2\sin\phi\cos\phi - 0\right) a_\rho + (2\rho z - 2z\sin^2\phi) a_\phi + \frac{1}{\rho}\left(2\rho\sin^2\phi - 0\right) a$$

(b) $$= 4z\sin\phi\cos\phi a_\rho + 2(\rho z - z\sin^2\phi) a_\phi + 2\sin^2\phi a_z$$

$$= 2z\sin 2\phi a_\rho + 2z(\rho - \sin^2\phi) a_\phi + 2\sin^2\phi a_z$$

$$\nabla x C = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(r\cos^2\theta\sin\theta\right] a_r - \frac{1}{r}\left[\frac{\partial}{\partial r}(r^2\cos^2\theta\right] a_\theta$$

(c) $$= \frac{r}{r\sin\theta}\left[(2\cos\theta)(-\sin\theta)\sin\theta + \cos\theta(\cos^2\theta)\right] a_r - \frac{\cos^2\theta}{r}(2r) a_\theta$$

$$= \frac{(\cos^3\theta - 2\sin^2\theta\cos\theta)}{\sin\theta} a_r - 2\cos^2\theta a_\theta$$

Prob. 3.39

Method 1: We can express **A** in spherical coordinates.

$$A = \frac{r}{r^3}\boldsymbol{a}_r = \frac{\boldsymbol{a}_r}{r^2},$$

$$\nabla \times A = \nabla \times \left(\frac{\boldsymbol{a}_r}{r^2}\right) = \nabla\left(\frac{1}{r^2}\right) \times \boldsymbol{a}_r = \frac{-2}{r^3}\boldsymbol{a}_r \times \boldsymbol{a}_r = \mathbf{0}$$

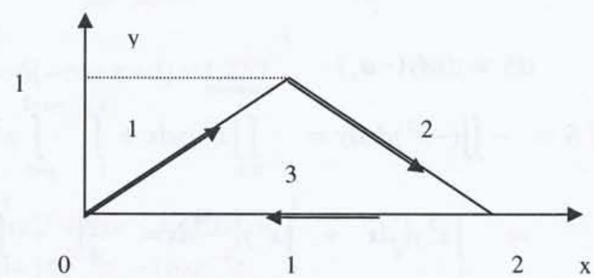
Method 2:

$$A = \frac{x}{r^3}\boldsymbol{a}_x + \frac{y}{r^3}\boldsymbol{a}_y + \frac{z}{r^3}\boldsymbol{a}_z$$

$$\nabla \times A = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{r^3} & \frac{y}{r^3} & \frac{z}{r^3} \end{vmatrix} = \left\{ -\frac{3}{2} z(x^2+y^2+z^2)^{-5/2}(2y) - -\frac{3}{2} y(x^2+y^2+z^2)^{-5/2}(2z) \right\} \boldsymbol{a}_x + \ldots$$

$$= \boldsymbol{0}$$

Prob. 3.40



(a)

$$\oint_L \boldsymbol{F} \bullet d\boldsymbol{l} = (\int_1 + \int_2 + \int_3) \boldsymbol{F} \bullet d\boldsymbol{l}$$

For 1, $y = x \quad dy = dx, d\bar{l} = dx\,\bar{a}_x + dy\,\bar{a}_y.$

$$\int_1 \boldsymbol{F} \bullet d\boldsymbol{l} = \int_0^1 x^3 dx - x dx = -\frac{1}{4}$$

For 2, $y = -x + 2, dy = -dx, d\bar{l} = dx\,\bar{a}_x + dy\,\bar{a}_y.$

$$\int_2 \boldsymbol{F} \bullet d\boldsymbol{l} = \int_1^2 (-x^3 + 2x^2 - x + 2) dx = \frac{17}{12}$$

For 3,

$$\int_3 \boldsymbol{F} \bullet d\boldsymbol{l} = \int_2^0 x^2 y dx \Big|_{y=0} = 0$$

$$\oint_L \boldsymbol{F} \bullet d\boldsymbol{l} = -\frac{1}{4} + \frac{17}{12} + 0 = \underline{\underline{\frac{7}{6}}}$$

(b)

$$\nabla \times \boldsymbol{F} = -x^2 \boldsymbol{a}_z \ ; \qquad d\boldsymbol{S} = dxdy(-\boldsymbol{a}_z)$$

$$\int (\nabla \times \boldsymbol{F}) \bullet d\boldsymbol{S} = -\iint (-x^2) dxdy = \int_0^1 \int_0^x x^2 dydx + \int_1^2 \int_{y=0}^{-x+2} x^2 dydx$$

$$= \int_0^1 x^2 y \Big|_0^x dx + \int_1^2 x^2 y \Big|_0^{-x+2} dx = \frac{x}{4}\Big|_0^1 + \int_1^2 x^2(-x+2) dx = \underline{\underline{\frac{7}{6}}}$$

(c) <u>Yes</u>

Prob. 3.41

$$\oint \boldsymbol{A} \bullet d\boldsymbol{l} = \int_{\rho=2}^1 \rho \sin\phi d\rho \Big|_{\phi=0} + \int_{\phi=0}^{\pi/2} \rho^2 \rho d\phi \Big|_{\rho=1} + \int_{\rho=1}^2 \rho \sin\phi d\rho \Big|_{\phi=90^o} + \int_{\phi=\pi/2}^0 \rho^3 d\phi \Big|_{\rho=2}$$

$$= \frac{\pi}{2} + \frac{1}{2}(4-1) + 8(-\frac{\pi}{2}) = \underline{\underline{-9.4956}}$$