Prob. 5.18

$$\rho_{pv} = -\nabla \bullet \boldsymbol{P} = -\frac{1}{\rho}\frac{\partial}{\partial \rho}(p_o \rho^2) = \underline{\underline{-2p_o}}$$

The surface polarization charge is

$$\rho_{ps} = \boldsymbol{P} \bullet \boldsymbol{a}_\rho \bigg|_{\rho = a} = \underline{\underline{p_o a}}$$

Prob. 5.19

(a)

$$Q_{s1} = \int_S \boldsymbol{P} \bullet d\boldsymbol{S}, d\boldsymbol{S} = r^2 \sin d\theta d\phi(-\boldsymbol{a}_r)$$

$$= -\iint 4r\, r^2 \sin\theta d\theta d\phi \bigg|_{r = 1.2cm}$$

$$= -4(1.2)^3 (10^{-6}) \int_0^{2\pi} d\phi \int \sin\theta d\theta (10^{-12})$$

$$= -6.912(2\pi)(2)\times 10^{-18}$$

$$= \underline{\underline{-86.86\times 10^{-18}\ \ \text{C}}}$$

(b)

$$Q_{s2} = \int_S \boldsymbol{P} \bullet d\boldsymbol{S}, d\boldsymbol{S} = r^2 \sin d\theta d\phi(-\boldsymbol{a}_r)$$

$$= -\iint 4r\, r^2 \sin\theta d\theta d\phi \bigg|_{r = 2.6cm}$$

$$= -4(2.6)^3 (10^{-6}) \int_0^{2\pi} d\phi \int \sin\theta d\theta (10^{-12})$$

$$= -4(2.6)^3 (2\pi)(2)\times 10^{-18} = \underline{\underline{883.5\times 10^{-18}\ \ \text{C}}}$$

(c)

$$\rho_{pv} = -\nabla \bullet \boldsymbol{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}(4r^3)\ \ \text{pC/m}^3 = -12\text{pC/m}^3$$

$$Q_v = \int_v \rho_{pv} dv = -12 \iiint dv = -12\int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \int_{1.2}^{2.6} r^2 dr (10^{-18})$$

$$= -12(2)(2\pi)\frac{r^3}{3}\bigg|_{1.2}^{2.6}(10^{-18}) = -16\pi(2.6^3 - 1.2^3)(10^{-18})$$

$$= \underline{\underline{-796.61\times 10^{-18}\ \ \text{C}}}$$

Prob. 5.20

$$\boldsymbol{D} = \varepsilon_o \varepsilon_r \boldsymbol{E} = 2.1x\frac{10^{-9}}{36\pi}(6,12,-20) = \underline{\underline{0.1114\boldsymbol{a}_x + 0.2228\boldsymbol{a}_y - 0.3714\boldsymbol{a}_z \text{ nC/m}^2}}$$

$$\boldsymbol{P} = \chi_e \varepsilon_o \boldsymbol{E} = 1.1x\frac{10^{-9}}{36\pi}(6,12,-20) = \underline{\underline{0.0584\boldsymbol{a}_x + 0.1167\boldsymbol{a}_y - 0.1945\boldsymbol{a}_z \text{ nC/m}^2}}$$

Prob. 5.21

$$\boldsymbol{E} = -\nabla V = -\left(\frac{\partial V}{\partial \rho}\boldsymbol{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\boldsymbol{a}_\phi + \frac{\partial V}{\partial z}\boldsymbol{a}_z\right)$$

$$= \left(10z\sin\phi \boldsymbol{a}_\rho + 10z\cos\phi \boldsymbol{a}_\phi + 10\rho\sin\phi \boldsymbol{a}_z\right)$$

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} = 5\varepsilon_o \boldsymbol{E}$$

$$= \underline{\underline{-50\varepsilon_0\left(z\sin\phi \boldsymbol{a}_\rho + z\cos\phi \boldsymbol{a}_\phi + \rho\sin\phi \boldsymbol{a}_z\right) \text{ C/m}^2}}$$

Prob. 5.22

(a) $\boldsymbol{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\boldsymbol{a}_x + \frac{\partial V}{\partial y}\boldsymbol{a}_y + \frac{\partial V}{\partial z}\boldsymbol{a}_z\right) = \underline{\underline{-20xyz\boldsymbol{a}_x - 10x^2z\boldsymbol{a}_y - 10(x^2y - z)\boldsymbol{a}_z \text{ V/m}}}$

(b) $\boldsymbol{D} = \varepsilon \boldsymbol{E} = 5\varepsilon_o \boldsymbol{E} = \underline{\underline{-0.8842xyz\boldsymbol{a}_x - 0.4421x^2z\boldsymbol{a}_y - 0.4421(x^2y - z)\boldsymbol{a}_z \text{ nC/m}^2}}$

(c) $\boldsymbol{P} = \chi_e \varepsilon_o \boldsymbol{E} = 4\varepsilon_o \boldsymbol{E} = \underline{\underline{-0.7073xyz\boldsymbol{a}_x - 0.3537x^2z\boldsymbol{a}_y - 0.3537(x^2y - z)\boldsymbol{a}_z \text{ nC/m}^2}}$

(d) $\rho_v = -\varepsilon\nabla^2 V$

$$\nabla^2 V = \frac{\partial}{\partial x}(20xyz) + \frac{\partial}{\partial y}(10x^2z) + \frac{\partial}{\partial z}(10x^2y - 10z) = 20yz - 10$$

$$\rho_v = -5\varepsilon_o 10(2yz - 1) = \underline{\underline{-0.8854yz + 0.4427 \text{ nC/m}^3}}$$

Prob. 5.23

$$\boldsymbol{P} = \chi_e \varepsilon_o \boldsymbol{E} = \chi_e \varepsilon_o \frac{\boldsymbol{D}}{\varepsilon_o \varepsilon_r} = \frac{(\varepsilon_r - 1)\boldsymbol{D}}{\varepsilon_r} = \frac{1.4}{2.4} \times 450\boldsymbol{a}_x \text{ nC/m}^2$$

$$\underline{\underline{\boldsymbol{P} \quad = 262.5\boldsymbol{a}_x \text{ nC/m}^2}}$$

Prob. 5.24 (a) Applying Coulomb's law,

$$E_r = \begin{cases} \dfrac{D_r}{\varepsilon_o} = \dfrac{Q}{4\pi\varepsilon_o r^2}, & r > b \\ \dfrac{D_r}{\varepsilon} = \dfrac{Q}{4\pi\varepsilon r^2}, & a < r < b \end{cases}$$

$$P = \frac{\varepsilon_r - 1}{\varepsilon_r} D \qquad (= D - \varepsilon_o E)$$

Hence

$$P_r = \frac{\varepsilon_r - 1}{\varepsilon_r} \cdot \frac{Q}{4\pi r^2}, \qquad a < r < b$$

(b) $$\rho_{pv} = -\nabla \bullet \mathbf{P} = -\frac{1}{r^2}\frac{d}{dr}(r^2 P_r) = 0$$

(c)

$$\rho_{ps} = \mathbf{P} \bullet (-\mathbf{a}_r) = -\frac{Q}{4\pi a^2}(\frac{\varepsilon_r - 1}{\varepsilon_r}), \qquad r = a$$

$$\rho_{ps} = \mathbf{P} \bullet (\mathbf{a}_r) = -\frac{Q}{4\pi b^2}(\frac{\varepsilon_r - 1}{\varepsilon_r}), \qquad r = b$$

Prob. 5.25

$$F_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_o d^2} = 2.6 \text{ nN}, \qquad F_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_o \varepsilon_r d^2} = 1.5 \text{ nN}$$

$$\frac{F_1}{F_2} = \frac{2.6}{1.5} = \varepsilon_r = 1.733$$

Prob. 5.26

(a) By Gauss's law,

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q_{enc} \longrightarrow D_r = \frac{Q}{4\pi r^2}$$

$$E_r = \frac{D_r}{\varepsilon} = \frac{Q}{4\pi\varepsilon r^2}$$

$$W = \int_v \frac{1}{2}\varepsilon |\mathbf{E}|^2 \, dv, \qquad dv = r^2 \sin\theta dr d\phi d\theta$$

$$W = \frac{1}{2}\varepsilon \int_{\phi=0}^{2\pi}\int_{\theta}^{\pi}\int_{r=a}^{\infty} \frac{Q^2}{16\pi^2\varepsilon^2 r^4} r^2 \sin\theta dr d\phi d\theta = \frac{Q^2}{8\pi\varepsilon a}$$

(b) D_r remains the same but

$$E_r = \frac{D_r}{\varepsilon} = \frac{Q}{4\pi r^2 \varepsilon_o \left(1+\frac{a}{r}\right)^2} = \frac{Q}{4\pi\varepsilon_o (r+a)^2}$$

$$W = \int_v \frac{1}{2}\varepsilon |E|^2\, dv = \frac{1}{2}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=a}^{\infty} \frac{Q^2 r^2 \sin\theta\, dr\, d\theta\, d\phi}{16\pi^2\varepsilon^2 (r+a)^4}\varepsilon_o\left(\frac{r+a}{r}\right)^2$$

$$= \frac{Q^2}{32\pi^2\varepsilon_o}(4\pi)\int_a^{\infty}\frac{dr}{(r+a)^2} = \frac{Q^2}{8\pi\varepsilon_o}\left(-\frac{1}{r+a}\Big|_a^{\infty}\right) = \frac{Q^2}{8\pi\varepsilon_o}\frac{1}{2a}$$

$$W = \underline{\underline{\frac{Q^2}{16a\pi\varepsilon_o}}}$$

Prob. 5.27

(a)

$$\rho_v = \begin{cases} \rho_o, & 0 < r < a \\ 0, & r > a \end{cases}$$

For r < a, $\varepsilon E_r (4\pi r^2) = \rho_o \frac{4\pi r^3}{3} \longrightarrow E_r = \frac{\rho_o r}{3\varepsilon}$

$$V = -\int \mathbf{E} \bullet d\mathbf{l} = -\frac{\rho_o r^2}{6\varepsilon} + c_1$$

For r > a, $\varepsilon_o E_r (4\pi r^2) = \rho_o \frac{4\pi a^3}{3} \longrightarrow E_r = \frac{\rho_o a^3}{3\varepsilon_o r^2}$

$$V = -\int E \bullet dl = \frac{\rho_o a^3}{3\varepsilon_o r} + c_2$$

As $r \longrightarrow \infty$, $V = 0$ and $c_2 = 0$

At r = a, $V(a^+) = V(a^-)$

$$-\frac{\rho_o a^2}{6\varepsilon_o\varepsilon_r} + c_1 = \frac{\rho_o a^2}{3\varepsilon_o} \longrightarrow c_1 = \frac{\rho_o a^2}{6\varepsilon_o\varepsilon_r}(2\varepsilon_r + 1)$$

$$V(r=0) = c_1 = \underline{\underline{\frac{\rho_o a^2 (2\varepsilon_r + 1)}{6\varepsilon_o\varepsilon_r}}}$$

(b) $V(r=a) = \underline{\underline{\frac{\rho_o a^2}{3\varepsilon_o}}}$

$$\rho_{vo} = \frac{Q}{V} = \frac{1}{\frac{4\pi}{3}\times10^{-6}\times8} = \underline{\underline{29.84 \text{ kC/m}^3}}$$

$$\rho_v = \rho_{vo}e^{-t/T_r} = 29.84e^{-2/4.42} = \underline{\underline{18.98 \text{ kC/m}^3}}$$

Prob. 5.36

$$P_1 = \chi_{e1}\varepsilon_o E_1 = \chi_{e1}\varepsilon_o \frac{D_1}{\varepsilon_o\varepsilon_{r1}} = \frac{4-1}{4}D_1 = \frac{3}{4}D_1$$

$$= \underline{\underline{12\mathbf{a}_x + 22.5\mathbf{a}_y - 15\mathbf{a}_z \text{ nC/m}^2}}$$

$$D_{2n} = D_{1n} = -20\mathbf{a}_z$$

$$E_{2t} = E_{1t} \longrightarrow \frac{D_{2t}}{\varepsilon_2} = \frac{D_{1t}}{\varepsilon_1}$$

$$D_{2t} = \frac{\varepsilon_2}{\varepsilon_1}D_{1t} = \frac{6.5\varepsilon_o}{4\varepsilon_o}(16\mathbf{a}_x + 30\mathbf{a}_y) = 26\mathbf{a}_x + 48.75\mathbf{a}_y$$

$$D_2 = D_{2n} + D_{2t} = \underline{\underline{26\mathbf{a}_x + 48.75\mathbf{a}_y - 20\mathbf{a}_z \text{ nC/m}^2}}$$

Prob. 5.37

Let $x > 0$ be region 1 and $x < 0$ be region 2.

$$D_{1n} = 50\mathbf{a}_x, \quad D_{1t} = 80\mathbf{a}_y - 30\mathbf{a}_z$$

$$D_{2n} = D_{1n} = 50\mathbf{a}_x$$

$$E_{2t} = E_{1t} \longrightarrow \frac{D_{2t}}{\varepsilon_2} = \frac{D_{1t}}{\varepsilon_1}$$

$$D_{2t} = \frac{\varepsilon_2}{\varepsilon_1}D_{1t} = \frac{7.6}{2.1}(80\mathbf{a}_y - 30\mathbf{a}_z) = 289.5\mathbf{a}_y - 108.6\mathbf{a}_z$$

$$D_2 = D_{2t} + D_{2n} = \underline{\underline{50\mathbf{a}_x + 289.5\mathbf{a}_y - 108.6\mathbf{a}_z \text{ nC/m}^2}}$$

Prob. 5.38

f(x,y)= 4x +3y –10=0

$$\nabla f = 4\mathbf{a}_x + 3\mathbf{a}_y \longrightarrow \mathbf{a}_n = -\frac{\nabla f}{|\nabla f|} = \frac{-(4\mathbf{a}_x + 3\mathbf{a}_y)}{5} = -0.8\mathbf{a}_x - 0.6\mathbf{a}_y$$

The minus sign is chosen for $\mathbf{a}_n$ because it is directed toward the origin.

$$D_{1n} = (D_1 \cdot \mathbf{a}_n)\mathbf{a}_n = (1.6 - 2.4)\mathbf{a}_n = -0.64\mathbf{a}_x - 0.48\mathbf{a}_y$$

$$D_{1t} = D_1 - D_{1n} = 2.64\mathbf{a}_x - 3.52\mathbf{a}_y + 6.5\mathbf{a}_z$$

$$D_{2n} = D_{1n} = -0.64\mathbf{a}_x - 0.48\mathbf{a}_y$$

$$E_{2t} = E_{1t} \longrightarrow \frac{D_{2t}}{\varepsilon_2} = \frac{D_{1t}}{\varepsilon_2}$$

$$E_1 = D_1 / \varepsilon_1 = \frac{(12,-14,21)\times 10^{-9}}{3.5\times\frac{10^{-9}}{36\pi}} = 387.8\mathbf{a}_\rho - 452.4\mathbf{a}_\phi + 678.6\mathbf{a}_z$$

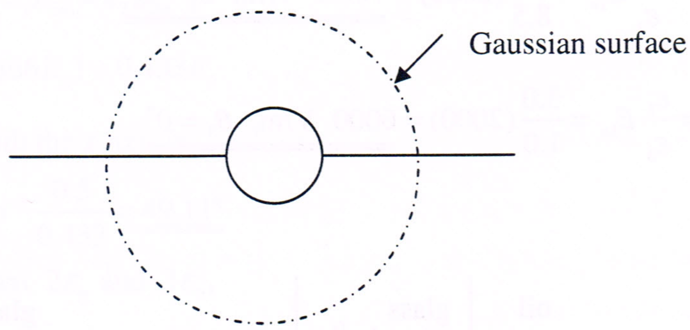
(b) $$\mathbf{P}_2 = \varepsilon_o \chi_{e2} \mathbf{E}_2 = 0.5\varepsilon_o \frac{\mathbf{D}_2}{\varepsilon_2} = \frac{0.5\varepsilon_o}{1.5\varepsilon_o}(12,-6,9) = \underline{\underline{4\mathbf{a}_\rho - 2\mathbf{a}_\phi + 3\mathbf{a}_z \text{ nC/m}^2}}$$

$$\rho_{v2} = \nabla \bullet \mathbf{P}_2 = 0$$

(c) $$w_{E1} = \frac{1}{2}\mathbf{D}_1 \bullet \mathbf{E}_1 = \frac{1}{2}\frac{\mathbf{D}_1 \bullet \mathbf{D}_1}{\varepsilon_o \varepsilon_{r1}} = \frac{1}{2}\frac{(12^2+14^2+21^2)x10^{-18}}{3.5x\frac{10^{-9}}{36\pi}} = \underline{\underline{12.62 \ \mu\text{J/m}^2}}$$

$$w_{E2} = \frac{1}{2}\frac{\mathbf{D}_2 \bullet \mathbf{D}_2}{\varepsilon_o \varepsilon_{r2}} = \frac{1}{2}\frac{(12^2+6^2+9^2)x10^{-18}}{1.5x\frac{10^{-9}}{36\pi}} = \underline{\underline{9.839 \ \mu\text{J/m}^2}}$$

Prob. 5.41



$$Q = \int \mathbf{D}.dS = \varepsilon_1 E_r \frac{4\pi r^2}{2} + \varepsilon_2 E_r \frac{4\pi r^2}{2} = 2\pi r^2(\varepsilon_1+\varepsilon_2)E_r$$

$$\underline{\underline{E_r = \begin{cases} \dfrac{Q}{2\pi(\varepsilon_1+\varepsilon_2)r^2}, & r > a \\ 0, & r < a \end{cases}}}$$

P 5.46

$$\vec{E}_1 = 100\,\vec{a}_x + 300\,\vec{a}_y - 50\,\vec{a}_z$$

$$\vec{a}_n = \vec{a}_x$$

$$\vec{E}_{1n} = (\vec{E}_1 \cdot \vec{a}_x)\vec{a}_x = 100\,\vec{a}_x$$

$$\vec{E}_{1t} = \vec{E}_1 - \vec{E}_{1n} = 300\,\vec{a}_y - 50\,\vec{a}_z$$

$$B \cdot C \Rightarrow \vec{E}_{1t} = \vec{E}_{2t}$$

$$\Leftrightarrow \vec{E}_{2t} = 300\,\vec{a}_y - 50\,\vec{a}_z$$

$$B \cdot C \Rightarrow (\vec{D}_1 - \vec{D}_2) \cdot \vec{a}_{n21} = \rho_s \quad \text{but } \vec{a}_{n21} = -\vec{a}_x$$

$$\Leftrightarrow -\vec{D}_1 \cdot \vec{a}_x + \vec{D}_2 \cdot \vec{a}_x = \rho_s$$

$$\Leftrightarrow -\varepsilon_1 \vec{E}_1 \cdot \vec{a}_x + \vec{D}_2 \cdot \vec{a}_x = \rho_s$$

$$\Leftrightarrow -100\varepsilon_0 + D_{2n} = \rho_s$$

$$\Leftrightarrow D_{2n} = \rho_s + 100\varepsilon_0$$

$$\vec{D}_{2n} = D_{2n}\,\vec{a}_x = (\rho_s + 100\varepsilon_0)\vec{a}_x$$

$$\vec{E}_{2n} = \frac{\vec{D}_{2n}}{\varepsilon_2} = \frac{1}{4\varepsilon_0}(\rho_s + 100\varepsilon_0)\,\vec{a}_x$$

$$\vec{E}_2 = \vec{E}_{2n} + \vec{E}_{2t} = \left(25 + \frac{\rho_s}{4\varepsilon_0}\right)\vec{a}_x + 300\,\vec{a}_y - 50\,\vec{a}_z$$

$$= 39.12\,\vec{a}_x + 300\,\vec{a}_y - 50\,\vec{a}_z$$

$$\vec{D}_2 = \varepsilon_2 \vec{E}_2 = 4\varepsilon_0\,(39.12\,\vec{a}_x + 300\,\vec{a}_y - 50\,\vec{a}_z)$$