

$$E_{1n} = \frac{\epsilon_{r2}}{\epsilon_{r1}} E_{2n} = \frac{1}{2.5} 12 \cos 60^\circ = 2.4$$

$$E_1 = \sqrt{E_{1t}^2 + E_{1n}^2} = \underline{\underline{10.67}}$$

$$\tan \theta_1 = \frac{\epsilon_{r1}}{\epsilon_{r2}} \tan \theta_2 = \frac{2.5}{1} \tan 60^\circ = 4.33 \quad \longrightarrow \quad \underline{\underline{\theta_1 = 77^\circ}}$$

Note that $\theta_1 > \theta_2$.

P. E. 5.10

$$D = \epsilon_0 E = \frac{10^{-9}}{36\pi} (60, 20, -30) \times 10^{-3} = \underline{\underline{0.531a_x + 0.177a_y - 0.265a_z}} \text{ pC/m}^2$$

$$\rho_s = D_n = |D| = \frac{10^{-9}}{36\pi} (10) \sqrt{36 + 4 + 9} (10^{-3}) = \underline{\underline{0.619}} \text{ pC/m}^2$$

Prob. 5.1

$$I = \int \mathbf{J} \cdot d\mathbf{S}, \quad d\mathbf{S} = dydz \mathbf{a}_x$$

$$\begin{aligned} I &= \iint e^{-x} \cos(4y) dydz \Big|_{x=2} = e^{-2} \int_0^{\pi/3} \cos(4y) dy \int_0^4 dz \\ &= 4e^{-2} \left(\frac{\sin 4y}{4} \Big|_0^{\pi/3} \right) = e^{-2} \left(\sin\left(\frac{4\pi}{3}\right) - 0 \right) = \underline{\underline{-0.1172 \text{ A}}} \end{aligned}$$

Prob. 5.2

Method 1:

$$\begin{aligned} I &= \int \mathbf{J} \cdot d\mathbf{S} = \iint \frac{10}{r} e^{-10^3 t} r^2 \sin \theta d\theta d\phi \Big|_{t=2ms, r=4m} \\ &= 10(4) e^{-10^3 \times 2 \times 10^{-3}} \int_{\theta=0}^{\pi} \sin \theta d\theta \int_{\phi=0}^{2\pi} d\phi = 40e^{-2} (2)(2\pi) = 160\pi e^{-2} \\ &= \underline{\underline{68.03 \text{ A}}} \end{aligned}$$

Method 2:

$$I = \int \mathbf{J} \cdot d\mathbf{S} = \iint \frac{10}{r} e^{-10^3 t} dS = \frac{10}{r} e^{-10^3 t} (4\pi r^2)$$

since r is constant on the surface.

$$I = 40\pi r e^{-2} = 160\pi e^{-2} = \underline{\underline{68.03 \text{ A}}}$$

Prob. 5.3

$$\begin{aligned} I &= \int \mathbf{J} \cdot d\mathbf{S} = \iint \frac{10}{\rho} \sin \phi \rho d\phi dz = 10 \int_0^5 dz \int_0^\pi \sin \phi d\phi \\ &= 10(5)(-\cos \phi) \Big|_0^\pi = \underline{\underline{100 \text{ A}}} \end{aligned}$$

Prob. 5.4

$$\begin{aligned} I &= \int \mathbf{J} \cdot d\mathbf{S} = 5 \int_{\rho=0}^a \int_{\phi=0}^{2\pi} e^{-10\rho} \rho d\phi d\rho = 5 \int_0^{2\pi} d\phi \int_{\rho=0}^a \rho e^{-10\rho} d\rho \\ &= 5(2\pi) \left(\frac{e^{-10\rho}}{100} (-10\rho - 1) \right) \Big|_0^a = \frac{10\pi}{100} [e^{-10a} (-10a - 1) - 1(-0 - 1)] \\ &= \frac{\pi}{10} [e^{-0.04} (-0.04 - 1) + 1] = \frac{\pi}{10} (0.00078) = \underline{\underline{244.7 \mu\text{A}}} \end{aligned}$$

Prob. 5.5

$$I = -\frac{dQ}{dt} = 3 \times 10^{-4} e^{-3t}$$

$$I(t=0) = \underline{\underline{0.3 \text{ mA}}}, \quad I(t=2.5) = 0.3 e^{-7.5} = \underline{\underline{166 \text{ nA}}}$$

Prob. 5.6

$$R = \frac{l}{\sigma S} \longrightarrow \sigma = \frac{l}{RS} = \frac{2 \times 10^{-2}}{10^6 (\pi)(4 \times 10^{-3})^2} = \underline{\underline{3.978 \times 10^{-4} \text{ S/m}}}$$

$$\text{Prob. 5.7 (a)} \quad R = \frac{l}{\sigma S} = \frac{8 \times 10^{-2}}{3 \times 10^4 \pi (25) 10^{-6}} = \frac{8}{75\pi} = \underline{\underline{33.95 \text{ m}\Omega}}$$

$$\text{(b)} \quad I = V/R = 9 \times \frac{75\pi}{8} = \underline{\underline{265.1 \text{ A}}}$$

Prob. 5.28

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \epsilon_o E_o \begin{bmatrix} 4 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$D_x = \epsilon_o E_o (4+1-1) = 4\epsilon_o E_o$$

$$D_y = \epsilon_o E_o (1+3-1) = 3\epsilon_o E_o$$

$$D_z = \epsilon_o E_o (1+1-2) = 0$$

$$\underline{\underline{D = \epsilon_o E_o (4a_x + 3a_y) \text{ C/m}^2}}$$

Prob. 5.29

Since $\frac{\partial \rho_v}{\partial t} = 0$, $\nabla \cdot \mathbf{J} = 0$ must hold.

(a) $\nabla \cdot \mathbf{J} = 6x^2y + 0 - 6x^2y = 0 \longrightarrow$ This is possible.

(b) $\nabla \cdot \mathbf{J} = y + (z+1) \neq 0 \longrightarrow$ This is not possible.

(c) $\nabla \cdot \mathbf{J} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (z^2) + \cos \phi \neq 0 \longrightarrow$ This is not possible.

(d) $\nabla \cdot \mathbf{J} = \frac{1}{r^2} \frac{\partial}{\partial r} (\sin \theta) = 0 \longrightarrow$ This is possible.

Prob. 5.30

$$\nabla \cdot \mathbf{J} = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = 2e^{-2y} \cos 2x - 2e^{-2y} \cos 2x + 1 = 1 = -\frac{\partial \rho_v}{\partial t}$$

Hence, $\frac{\partial \rho_v}{\partial t} = \underline{\underline{-1 \text{ C/m}^3 \text{ s}}}$

Prob. 5.31

(a) $\nabla \cdot \mathbf{J} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\frac{100}{\rho} \right) = -\frac{100}{\rho^3}$

$$-\frac{\partial \rho_v}{\partial t} = \nabla \cdot \mathbf{J} = -\frac{100}{\rho^3} \longrightarrow \underline{\underline{\frac{\partial \rho_v}{\partial t} = \frac{100}{\rho^3} \text{ C/m}^3 \cdot \text{s}}}$$

(b) At (0, 1, 0), $\mathbf{R} = \mathbf{a}_y$

$$d\mathbf{H} = \frac{4\mathbf{a}_x \times \mathbf{a}_y}{4\pi(1)^3} = \underline{\underline{0.3183\mathbf{a}_z \text{ A/m}}}$$

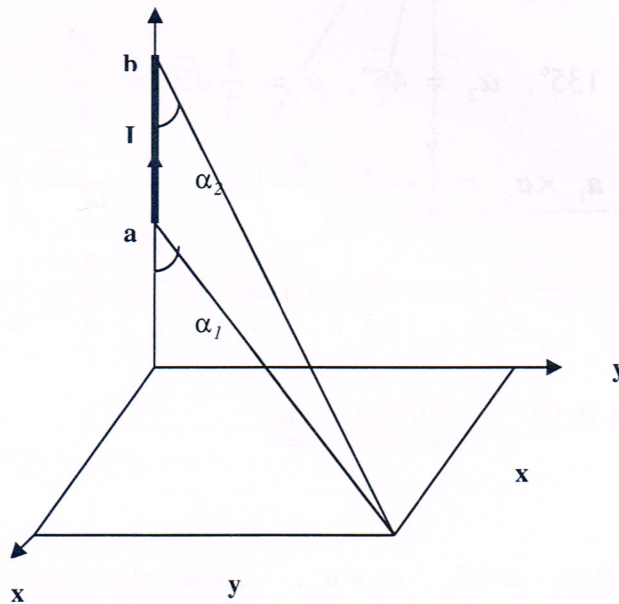
(c) At (0, 0, 1), $\mathbf{R} = \mathbf{a}_z$

$$d\mathbf{H} = \frac{4\mathbf{a}_x \times \mathbf{a}_z}{4\pi(1)^3} = \underline{\underline{-0.3183\mathbf{a}_y \text{ A/m}}}$$

(d) At (1, 1, 1), $\mathbf{R} = (1, 1, 1)$

$$d\mathbf{H} = \frac{4\mathbf{a}_x \times (\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)}{4\pi(3)^{3/2}} = \underline{\underline{61.26(-\mathbf{a}_y + \mathbf{a}_z) \text{ mA/m}}}$$

Prob. 7.5



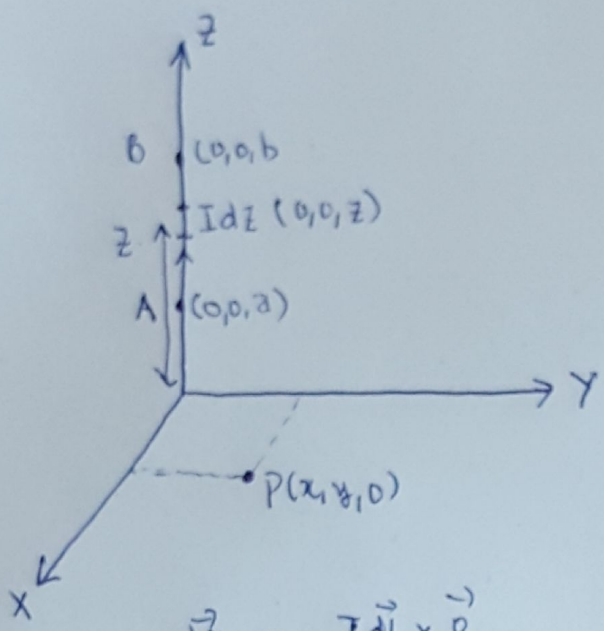
$$\mathbf{H} = \frac{I}{4\pi\rho} (\cos \alpha_2 - \cos \alpha_1) \mathbf{a}_\phi$$

$$\rho = \sqrt{x^2 + y^2}, \cos \alpha_1 = \frac{a}{\sqrt{a^2 + \rho^2}}, \cos \alpha_2 = \frac{b}{\sqrt{b^2 + \rho^2}}$$

$\mathbf{a}_\phi = \mathbf{a}_1 \times \mathbf{a}_\rho = \mathbf{a}_z \times \mathbf{a}_\rho = \mathbf{a}_\phi$. Hence,

$$\mathbf{H} = \frac{I}{4\pi\sqrt{x^2 + y^2}} \left[\frac{b}{\sqrt{x^2 + y^2 + b^2}} - \frac{a}{\sqrt{x^2 + y^2 + a^2}} \right] \mathbf{a}_\phi$$

7.5



$\vec{r} = (x, y, 0)$ - location of $d\vec{H}$
 $\vec{r}' = (0, 0, z)$ - location of $I d\vec{l}$

$$d\vec{H} = \frac{I d\vec{l} \times \vec{R}}{4\pi R^3} \quad I d\vec{l} = I dz \vec{a}_z$$

$$\vec{R} = \vec{r} - \vec{r}' = (x, y, 0) - (0, 0, z) = (x, y, -z)$$

$$R^3 = (x^2 + y^2 + z^2)^{3/2}$$

$$d\vec{H} = \frac{I dz \vec{a}_z \times (x\vec{a}_x + y\vec{a}_y + z\vec{a}_z)}{4\pi (x^2 + y^2 + z^2)^{3/2}}$$

converting to cylindrical coordinates

$$= \frac{I dz \vec{a}_z \times (\rho \vec{a}_\rho + z\vec{a}_z)}{4\pi (\rho^2 + z^2)^{3/2}}$$

$$\rho^2 = x^2 + y^2$$

$$d\vec{H} = \frac{I \rho dz a_\phi}{4\pi (\rho^2 + z^2)^{3/2}}$$

$$\Rightarrow \vec{H} = \int d\vec{H} = \frac{I \rho}{4\pi} \int_a^b \frac{\vec{a}_\phi dz}{(\rho^2 + z^2)^{3/2}} = \frac{I \rho}{4\pi} \left[\frac{z}{\rho^2 \sqrt{\rho^2 + z^2}} \right]_a^b \vec{a}_\phi$$

$$= \frac{\vec{a}_\phi I}{4\pi \rho} \left[\frac{b}{\sqrt{\rho^2 + b^2}} - \frac{a}{\sqrt{\rho^2 + a^2}} \right] = \frac{I}{4\pi \sqrt{x^2 + y^2}} \left[\frac{b}{\sqrt{x^2 + y^2 + b^2}} - \frac{a}{\sqrt{x^2 + y^2 + a^2}} \right] \vec{a}_\phi$$

$\rho = \sqrt{x^2 + y^2}$

(a) Consider the figure above.

$$\mathbf{AB} = (1, 1, 0) - (2, 0, 0) = (-1, 1, 0)$$

$$\mathbf{AC} = (0, 0, 5) - (2, 0, 0) = (-2, 0, 5)$$

$\mathbf{AB} \cdot \mathbf{AC} = 2$, i.e. \mathbf{AB} and \mathbf{AC} are not perpendicular.

$$\cos(180^\circ - \alpha_1) = \frac{\mathbf{AB} \cdot \mathbf{AC}}{|\mathbf{AB}||\mathbf{AC}|} = \frac{2}{\sqrt{2}\sqrt{29}} \rightarrow \cos \alpha_1 = -\sqrt{\frac{2}{29}}$$

$$\mathbf{BC} = (0, 0, 5) - (1, 1, 0) = (-1, -1, 5)$$

$$\mathbf{BA} = (1, -1, 0)$$

$$\cos \alpha_2 = \frac{\mathbf{BC} \cdot \mathbf{BA}}{|\mathbf{BC}||\mathbf{BA}|} = \frac{-1+1}{|\mathbf{BC}||\mathbf{BA}|} = 0$$

i.e. $\mathbf{BC} = \boldsymbol{\rho} = (-1, -1, 5)$, $\rho = \sqrt{27}$

$$\mathbf{a}_\phi = \mathbf{a}_1 \times \mathbf{a}_\rho = \frac{(-1, 1, 0)}{\sqrt{2}} \times \frac{(-1, -1, 5)}{\sqrt{27}} = \frac{(5, 5, 2)}{\sqrt{54}}$$

$$\mathbf{H}_2 = \frac{10}{4\pi\sqrt{27}} \left(0 + \sqrt{\frac{2}{29}}\right) \frac{(5, 5, 2)}{\sqrt{2}\sqrt{27}} = \frac{5}{2\pi\sqrt{29}} \cdot \frac{(5, 5, 2)}{27} \text{ A/m}$$

$$= \underline{\underline{27.37 \mathbf{a}_x + 27.37 \mathbf{a}_y + 10.95 \mathbf{a}_z \text{ mA/m}}}$$

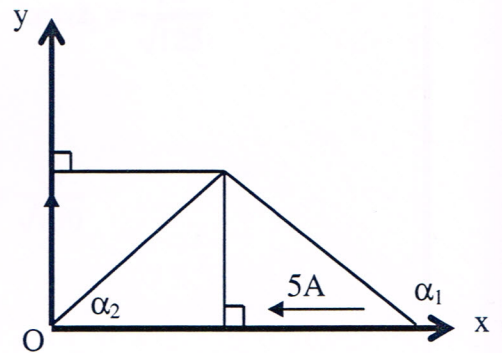
(b) $\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2 + \mathbf{H}_3 = (0, -59.1, 0) + (27.37, 27.37, 10.95)$
 $+ (-30.63, 30.63, 0)$

$$= \underline{\underline{-3.26 \mathbf{a}_x - 1.1 \mathbf{a}_y + 10.95 \mathbf{a}_z \text{ mA/m}}}$$

Prob. 7.9

(a) Let $\mathbf{H} = \mathbf{H}_x + \mathbf{H}_y = 2\mathbf{H}_x$

$$\mathbf{H}_x = \frac{I}{4\pi\rho} (\cos \alpha_2 - \cos \alpha_1) \mathbf{a}_\phi$$



where $\mathbf{a}_\phi = -\mathbf{a}_x \times \mathbf{a}_y = -\mathbf{a}_z$, $\alpha_1 = 180^\circ$, $\alpha_2 = 45^\circ$

$$\begin{aligned} \mathbf{H}_x &= \frac{5}{4\pi(2)} (\cos 45^\circ - \cos 180^\circ) (-\mathbf{a}_z) \\ &= \underline{\underline{-0.6792 \mathbf{a}_z \text{ A/m}}} \end{aligned}$$

(b) $\mathbf{H} = \mathbf{H}_x + \mathbf{H}_y$

where $\mathbf{H}_x = \frac{5}{4\pi(2)} (1-0) \mathbf{a}_\phi$, $\mathbf{a}_\phi = -\mathbf{a}_x \times -\mathbf{a}_y = \mathbf{a}_z$

$$= 198.9 \mathbf{a}_z \text{ mA/m}$$

$\mathbf{H}_y = 0$ since $\alpha_1 = \alpha_2 = 0$

$\mathbf{H} = \underline{\underline{0.1989 \mathbf{a}_z \text{ A/m}}}$

(c) $\mathbf{H} = \mathbf{H}_x + \mathbf{H}_y$

where $\mathbf{H}_x = \frac{5}{4\pi(2)} (1-0) (-\mathbf{a}_x \times \mathbf{a}_z) = 198.9 \mathbf{a}_y \text{ mA/m}$

$\mathbf{H}_y = \frac{5}{4\pi(2)} (1-0) (\mathbf{a}_y \times \mathbf{a}_z) = 198.9 \mathbf{a}_x \text{ mA/m}$

$\mathbf{H} = \underline{\underline{0.1989 \mathbf{a}_x + 0.1989 \mathbf{a}_y \text{ A/m}}}$

Prob. 7.10

For the side of the loop along y-axis,

$$\mathbf{H}_1 = \frac{I}{4\pi\rho} (\cos \alpha_2 - \cos \alpha_1) \mathbf{a}_\phi$$

where $\mathbf{a}_\phi = -\mathbf{a}_x$, $\rho = 2 \tan 30^\circ = \frac{2}{\sqrt{3}}$, $\alpha_2 = 30^\circ$, $\alpha_1 = 150^\circ$

$$\mathbf{H}_1 = \frac{5}{4\pi} \frac{\sqrt{3}}{2} (\cos 30^\circ - \cos 150^\circ) (-\mathbf{a}_x) = -\frac{15}{8\pi} \mathbf{a}_x$$

$\mathbf{H} = 3\mathbf{H}_1 = -1.79 \mathbf{a}_x \text{ A/m}$

(b) From Eq. (7.29),

$$H_{\phi} = \begin{cases} \frac{I\rho}{2\pi a^2}, & \rho < a \\ \frac{I}{2\pi\rho}, & \rho > a \end{cases}$$

At (0, 1 cm, 0),

$$H_{\phi} = \frac{3 \times 1 \times 10^{-2}}{2\pi \times 4 \times 10^{-4}} = \frac{300}{8\pi}$$

$$\underline{\underline{H = 11.94 a_{\phi} \text{ A/m}}}$$

At (0, 4 cm, 0),

$$H_{\phi} = \frac{3}{2\pi \times 4 \times 10^{-2}} = \frac{300}{8\pi}$$

$$\underline{\underline{H = 11.94 a_{\phi} \text{ A/m}}}$$

Prob. 7.22

For $0 < \rho < a$

$$\oint_L \mathbf{H} \cdot d\mathbf{l} = I_{enc} = \int \mathbf{J} \cdot d\mathbf{S}$$

$$H_{\phi} 2\pi\rho = \int_{\phi=0}^{2\pi} \int_{\rho=0}^{\rho} \frac{J_o}{\rho} \rho d\phi d\rho$$

$$= J_o 2\pi\rho$$

$$\underline{\underline{H_{\phi} = J_o}}$$

For $\rho > a$

$$\int \mathbf{H} \cdot d\mathbf{l} = \int \mathbf{J} \cdot d\mathbf{S} = \int_{\phi=0}^{2\pi} \int_{\rho=0}^a \frac{J_o}{\rho} \rho d\phi d\rho$$

$$H_{\phi} 2\pi\rho = J_o 2\pi a$$

$$H_{\phi} = \frac{J_o a}{\rho}$$

$$\underline{\underline{\text{Hence } H_{\phi} = \begin{cases} J_o, & 0 < \rho < a \\ \frac{J_o a}{\rho}, & \rho > a \end{cases}}}$$

Prob. 7.23

$$(a) \mathbf{J} = \nabla \times \mathbf{H} = \frac{1}{\rho} \frac{d}{d\rho} (\rho H_\phi) \mathbf{a}_z = \frac{1}{\rho} \frac{d}{d\rho} \left(k_o \frac{\rho^2}{a} \right) \mathbf{a}_z = \underline{\underline{\frac{2k_o}{a} \mathbf{a}_z}}$$

(b) For $\rho > a$,

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc} = \int \mathbf{J} \cdot d\mathbf{S} = \int_{\rho=0}^a \int_{\phi=0}^{2\pi} \frac{2k_o}{a} \rho d\rho d\phi = \frac{2k_o}{a} (2\pi) \frac{\rho^2}{2} \Big|_0^a$$

$$H_\phi 2\pi\rho = 2\pi k_o a \quad \longrightarrow \quad H_\phi = \frac{k_o a}{\rho}$$

$$\underline{\underline{\mathbf{H} = k_o \left(\frac{a}{\rho} \right) \mathbf{a}_\phi, \quad \rho > a}}$$

Prob. 7.24

$$\mathbf{J} = \nabla \times \mathbf{H} = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & x^2 & 0 \end{vmatrix} = (2x - 2y) \mathbf{a}_z$$

At (1, -4, 7), $x=1$, $y=-4$, $z=7$,

$$\mathbf{J} = [2(1) - 2(-4)] \mathbf{a}_z = \underline{\underline{10 \mathbf{a}_z \text{ A/m}^2}}$$

Prob. 7.25

(a)

$$\begin{aligned} \mathbf{J} = \nabla \times \mathbf{H} &= \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho H_\phi) \mathbf{a}_z = \frac{1}{\rho} \frac{\partial}{\partial \rho} (10^3 \rho^3) \mathbf{a}_z \\ &= \underline{\underline{3\rho \times 10^3 \mathbf{a}_z \text{ A/m}^2}} \end{aligned}$$

(b)

Method 1:

$$\begin{aligned} I &= \int_S \mathbf{J} \cdot d\mathbf{S} = \iint 3\rho \rho d\phi d\rho 10^3 = 3 \times 10^3 \int_0^2 \rho^2 d\rho \int_0^{2\pi} d\phi \\ &= 3 \times 10^3 (2\pi) \frac{\rho^3}{3} \Big|_0^2 = 16\pi \times 10^3 \text{ A} = \underline{\underline{50.265 \text{ kA}}} \end{aligned}$$

Method 2:

$$I = \oint_L \mathbf{H} \cdot d\mathbf{l} = 10^3 \int_0^{2\pi} \rho^2 \rho d\phi = 10^3 (8)(2\pi) = \underline{\underline{50.265 \text{ kA}}}$$

Prob. 7.46

$$\begin{aligned}
 \mathbf{B} = \nabla \times \mathbf{A} &= \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (A_\phi \sin \theta) - \frac{\partial A_\theta}{\partial \phi} \right] \mathbf{a}_r + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r} (r A_\phi) \right] \mathbf{a}_\theta \\
 &\quad + \frac{1}{r} \left[\frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right] \mathbf{a}_\phi \\
 &= \frac{1}{r \sin \theta} \frac{10}{r} 2 \sin \theta \cos \theta \mathbf{a}_r - \frac{1}{r} \frac{\partial}{\partial r} (10) \sin \theta \mathbf{a}_\theta + 0 \mathbf{a}_\phi \\
 \mathbf{B} &= \frac{20}{r^2} \cos \theta \mathbf{a}_r
 \end{aligned}$$

At $(4, 60^\circ, 30^\circ)$, $r = 4$, $\theta = 60^\circ$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_o} = \frac{1}{4\pi \times 10^{-7}} \left[\frac{20}{4^2} \cos 60^\circ \mathbf{a}_r \right] = \underline{\underline{4.974 \times 10^5 \mathbf{a}_r \text{ A/m}}}$$

Prob. 7.47

Applying Ampere's law gives

$$H_\phi \cdot 2\pi\rho = J_o \cdot \pi\rho^2$$

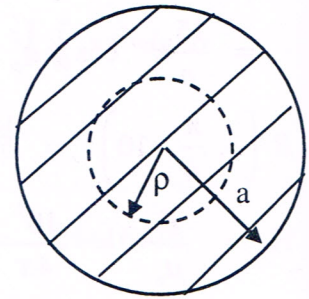
$$H_\phi = \frac{J_o}{2} \rho$$

$$B_\phi = \mu_o H_\phi = \mu_o \frac{J_o \rho}{2}$$

$$\text{But } \mathbf{B} = \nabla \times \mathbf{A} = -\frac{\partial A_z}{\partial \rho} \mathbf{a}_\phi + \dots$$

$$-\frac{\partial A_z}{\partial \rho} = \frac{1}{2} \mu J_o \rho \longrightarrow A_z = -\mu_o \frac{J_o \rho^2}{4}$$

$$\text{or } \mathbf{A} = \underline{\underline{-\frac{1}{4} \mu_o J_o \rho^2 \mathbf{a}_z}}$$

**Prob. 7.48**

$$\begin{aligned}
 \mathbf{B} = \nabla \times \mathbf{A} &= \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & A_z(x, y) \end{vmatrix} = \frac{\partial A_z}{\partial y} \mathbf{a}_x - \frac{\partial A_z}{\partial x} \mathbf{a}_y \\
 &= \underline{\underline{-\frac{\pi}{2} \sin \frac{\pi x}{2} \sin \frac{\pi y}{2} \mathbf{a}_x - \frac{\pi}{2} \cos \frac{\pi x}{2} \cos \frac{\pi y}{2} \mathbf{a}_y}}
 \end{aligned}$$

Prob. 7.49

$$\begin{aligned}
 \mathbf{B} &= \nabla \times \mathbf{A} = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\phi \sin \theta) \mathbf{a}_r - \frac{1}{r} \frac{\partial}{\partial r} (r A_\phi) \mathbf{a}_\theta \\
 &= \frac{1}{r \sin \theta} \frac{A_0}{r^2} (2 \sin \theta \cos \theta) \mathbf{a}_r - \frac{A_0}{r} \sin \theta (-r^{-2}) \mathbf{a}_\theta \\
 &= \frac{A_0}{r^3} (2 \cos \theta \mathbf{a}_r + \sin \theta \mathbf{a}_\theta)
 \end{aligned}$$

Prob. 7.50

$$(a) \quad \mathbf{J} = \nabla \times \mathbf{H} = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy^2 & x^2z & -y^2z \end{vmatrix} = (-2yz - x^2) \mathbf{a}_x + (2xz - 2xy) \mathbf{a}_z$$

At (2, -1, 3), $x=2$, $y=-1$, $z=3$.

$$\mathbf{J} = \underline{\underline{2\mathbf{a}_x + 16\mathbf{a}_z}} \text{ A/m}^2$$

$$(b) \quad -\frac{\partial \rho_v}{\partial t} = \nabla \cdot \mathbf{J} = 0 - 2x + 2x = 0$$

At (2, -1, 3),

$$\frac{\partial \rho_v}{\partial t} = \underline{\underline{0 \text{ C/m}^3\text{s}}}$$

Prob. 7.51

$$(a) \quad \mathbf{B} = \nabla \times \mathbf{A}$$

$$\begin{aligned}
 &= \left[\frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right] \mathbf{a}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right] \mathbf{a}_\phi + \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} (\rho A_\phi) - \frac{\partial A_\rho}{\partial \phi} \right] \mathbf{a}_z \\
 &= -\frac{\partial A_z}{\partial \rho} \mathbf{a}_\phi = 20 \rho \mathbf{a}_\phi \text{ } \mu\text{Wb/m}^2
 \end{aligned}$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} = \underline{\underline{\frac{-20\rho}{\mu_0} \mathbf{a}_\phi}} \text{ } \mu\text{A/m}$$

$$\begin{aligned}
 \mathbf{J} = \nabla \times \mathbf{H} &= \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\phi) \mathbf{a}_z \\
 &= \frac{1}{\mu_0 \rho} (-40\rho) \mathbf{a}_z = \underline{\underline{\frac{-40}{\mu_0} \mathbf{a}_z}} \text{ } \mu\text{A/m}^2
 \end{aligned}$$