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Elimination of fast mode in hydrodynamics of superfluid turbulence.

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Using the multi-time scales method we fulfill procedure of a separation of a fast and slow processes in the equations of HST. It is shown that slow stage of the evolution of transient heat load of moderate intensity obeys to the well-known nonlinear heat conductivity equation.

1. Introduction

In the presence of vortex tangle (VT), or superfluid turbulence (ST) (See e.g. [1]) any flow (or counterflow) of the superfluid turbulent HeII should be studied on base of equations of Hydrodynamics of Superfluid Turbulence (HST). HST as well as the methods of investigation of some of the problems are reviewed in [2]. The set of equations of HST is very cumbersome therefore an investigation of any important case requires numerical methods. However numerical simulation of the relevant problems of nonstationary flow or/and counterflow faces one serious obstacle. The point is that the HST which unify usual equations of hydrodynamics of superfluid helium and the Vinen equation for evolution of the vortex line density (VLD) has initially hyperbolic character. As a result the slow variation of hydrodynamic variables due to Gorter-Mellink force is accompanied by fast processes connected with propagation (and possible reflections) of the second sound. If one is only interested in slow variation of the field of the temperature, velocities and (VLD) the details of propagation and manifold reflections of the second sound is a excess information, which requires a huge numerical resource. Therefore it seems to be attractive to exclude the fast modes by pure analytical methods. In the present work we carry out the separation of slow and fast processes using the well known method of multi-time scales of the asymptotic theory of nonlinear equations [3].

2. Multi-time scale method

We will study a nonstationary counterflow of HeII of the moderate intensity in channel of length L . We will restrict ourselves by the quasi-one-dimensional cases, i.e. either pure one-dimensional one or cylindrical or spherical geometries. Dimensional analysis of the equations of HST shows that among of dimensionless criteria there is the criterion so called Strouhal number Sh which has a sense of the ratio between a decrement of damping the counterflow due to interaction with vortex tangle and the inverse time of the flight of the heat pulse. Depending on input parameters of the problem stated the number Sh can be either large or small. In these and only in these two limiting cases it is possible to realize effectively the procedure of separation of the slow and fast processes. Numerical estimations that for moderate heat loads the Strouhal number is large

$$Sh >> 1. \tag{1}$$

We will use this condition for effective separation of the processes with characteristic time $t_0 \sim L/c_2$ (second sound behavior) and of the processes with characteristic time $t_1 = L/(c_2 Sh)$ (evolution due to counterflow-vortex tangle mutual friction).

Pursuing the goal to separate the fast and slow processes in the equations of the HST we use the multi- times scales method of theory of perturbations. Following this method we introduce different scales of times

$$t_0' = t_0; \qquad t_1 = \epsilon t; \qquad t_2 = \epsilon^2 t \qquad .... \tag{2}$$

Where $\epsilon = 1/Sh << 1$. We are seeking for solution of set of the HST equations in form of asymptotic series

$$\begin{aligned} V_n' &= V_0'(x', t_0, t_1, t_2) + \epsilon V_1'(x', t_0, t_1, t_2) + ... \\ T' &= T_0'(x', t_0, t_1, t_2) + \epsilon T_1'(x', t_0, t_1, t_2) + .. \\ .L &= L_0'(x', t_0, t_1, t_2) + \epsilon L_1'(x', t_0, t_1, t_2) + ... \end{aligned} \tag{3}$$

The coefficients in series for normal velocity V_i', temperature T_i and (VLD) L_i are supposed to be of order of unit. These variables as well as variables x' and t' are made dimensionless by usual way (see e.g. [5]) The following step in studying of slow evolution of heat pulse is in substituting of multi-time scales series (2)(3) into equations of HST (see [1]). Gathering terms of the same order of smallness we obtain a chain of equations which lead to divergent (secular) solutions. Canceling step by step these secularities we obtain sets of equations of different orders of smallness (in parameter ε) governing different stages of evolution of the fields of temperature, counterflow velocities and (VLD). Studying the zero order we conclude that a first (fast) stage is propagation of heat pulses by the second sound mechanism which is subjected to strong attenuation due to interaction with superfluid turbulence. Analysis of the first order equations shows that after some period the second sound mode degenerated and further evolution of all of fields obey to other equations, which have a parabolic character unlike initial hyperbolic equations.

3. The Dresner nonlinear heat conductivity equation.

Let us consider the first ϵ^1 approach in more details. Here we will restrict ourselves by the plain case. The set of dimensionless equations for the dimensional normal velocity, temperature and VLD

$$\frac{\partial V_0^3}{\partial t_1'} = \frac{\partial^2 V_0}{\partial x'^2}; \quad \frac{\partial T_0}{\partial t_1'} + \frac{\partial V_0}{\partial x'} = 0 \qquad (4)$$

$$L_0/L_\infty = V_0^2$$

Here V_0, T_0, L_0 are zero terms in series (3) for dimensional normal velocity, temperature and VLD. Furthermore, excluding quantity V_0 from the set of equations written above we arrive at the following relation

$$\frac{\partial T_0}{\partial t_1'} = \frac{\partial}{\partial x'}\left(\frac{\partial T_0}{\partial x'}\right)^{1/3}. \qquad (5)$$

Relation (5) coincides with the widely used nonlinear heat-conductivity equation derived by Dresner (see [4]). In this connection it is worth discussing the method used by Dresner. He started with Gorter-Mellink relation which in our (dimensionless) notation takes a form

$$\frac{\partial T'}{\partial x'} = V'^3_n. \qquad (6)$$

Remembering that heat flux q is related with normal velocity V_n by relation $q = STV_n$ and using the energy conservation law Dresner derived equation similar to the equation (5). This method however is not correct from point of view of the full set of equations of HST. Indeed the Gorter-Mellink relations corresponds only to steady flow and in the nonstationary case one have to add the term $\partial V_n'/\partial t'$ in l.h.s. of Eq. (6). But neglecting this term implies that $V_n' = V_n'(x)$ which in turn implies that the temperature T' is also function of only x and does not change in time. In our approach $\partial V_0'/\partial t_1' \neq 0$ and there is no contradiction. It shows that regime of the nonlinear heat-conductivity equation takes place only for "slow" time and validity of overall procedure requires a fulfillment of conditions 1. To conclude this chapter we would like to point out the region of fulfillment of the condition (1) in terms of heat flux and of sizes of channels. Using the definition of the Sh number and thermodynamic parameters we obtain that in the temperature region $T = 1.4 \div 2.1$K condition (1) is equivalent to the following one:

$$q^2 L \; >> \; 0.2 \div 0.6 \; W/cm^2. \qquad (7)$$

Thus for heat load of moderate intensity $q = 1 \div 10$ W/cm^2 and for sizes of the channels $L = 10 \div 10^2 cm$ relation (1) and consequently (7) are valid with a good accuracy.

4. Conclusion

We described a procedure of a separation of a fast and slow processes in the equations of HST. As an illustration of the procedure developed we studied a slow stage of the evolution of transient heat load of moderate intensity, showed that it coincided with the Dresner description and found a range of its validity.

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