Spectral anomalies in a Fraunhofer diffraction pattern

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Received March 29, 2002

We show that spectacular spectral changes take place in the vicinity of the dark rings of the Airy pattern formed with spatially coherent, polychromatic light diffracted at a circular aperture.

OCIS codes: 260.1960, 260.3160, 300.6170.

Phase singularities of optical waves are the points where the intensity of a monochromatic wave field has zero value, and, hence, where the phase of the field is indeterminate. It has been known for some time that in the neighborhood of such phase singularities, the wave field has a rich structure exhibiting, for example, dislocations and optical vortices.[1,2] Studies of these special regions have gradually developed into a new branch of physical optics, sometimes referred to as *singular optics.*[3]

In a recent paper,[4] it was shown that drastic spectral changes take place in the vicinity of intensity zeros in the focal region of a polychromatic, spatially coherent, converging spherical wave.[4] In this Letter, we demonstrate that similar spectral changes occur in the neighborhood of some special directions, which we call *critical directions,* associated with the dark rings of the Airy pattern in the Fraunhofer region of a field produced by diffraction of a spatially coherent, *polychromatic* wave at a circular aperture. These results indicate that spectral anomalies are not restricted to the special fields considered in Ref. 4 but are generic of spatially coherent polychromatic wave fields in regions where a spectral component of the light has zero value.

Consider a spatially coherent polychromatic plane wave, with spectral density $S^{(i)}(\omega)$, incident normally upon a circular aperture $\mathcal{A}$ of radius a in an opaque screen (see Fig. 1). It follows from elementary theory of Fraunhofer diffraction (Ref. 5, Sec. 8.5.2) that the spectrum of the diffracted light at a point P, in a direction that makes the angle θ with the normal to the aperture plane $z = 0$, is given, apart from a constant proportionality factor, by the expression

$$S(r, \theta, \omega) \propto \left(\frac{\omega}{r}\right)^2 \left[\frac{2J_1(ka \sin \theta)}{ka \sin \theta}\right]^2 S^{(i)}(\omega). \qquad (1)$$

Here $k = 2\pi/\lambda = \omega/c$ is the wave number associated with the frequency ω, r is the distance from the center of the aperture to the point P, and $J_1(x)$ is the Bessel function of the first kind and of the first order.

Suppose that the spectrum of the incident field consists of a single line of a Gaussian profile, $S^{(i)}(\omega) \propto \exp[-(\omega - \omega_0)^2/2\sigma^2]$. The analysis of Eq. (1) reveals a peculiar behavior of the spectrum of the diffracted light in the vicinity of any direction in which the intensity of the diffracted wave has zero value at any frequency ω within the spectrum. Such critical directions θ_c are associated with the positions of the dark Airy rings at that frequency. As is well known and as follows at once from Eq. (1), the first dark Airy ring is in the direction

$$\theta_{c1} \simeq 0.61\lambda_0/a. \qquad (2)$$

In Fig. 2, the spectrum of the incident wave is shown, and in Fig. 3, the spectra of the diffracted light are displayed, in the lowest-order (first) critical direction and in two directions very close to it. We see that in the critical direction, the spectral line is split into two lines, whereas in the immediate neighborhood of that critical direction, it is either blueshifted or redshifted. Physically, such a behavior of the spectrum is a consequence of interference of different spectral components of the light that is diffracted by the aperture. It follows from the analysis of Eq. (1) that such spectral changes occur in the very narrow proximity $\Delta\theta \sim 10^{-2} \times \theta_{c1}$ of the critical direction. Further, it is clear from Eq. (1) that the spectrum remains invariant, up to a scaling factor

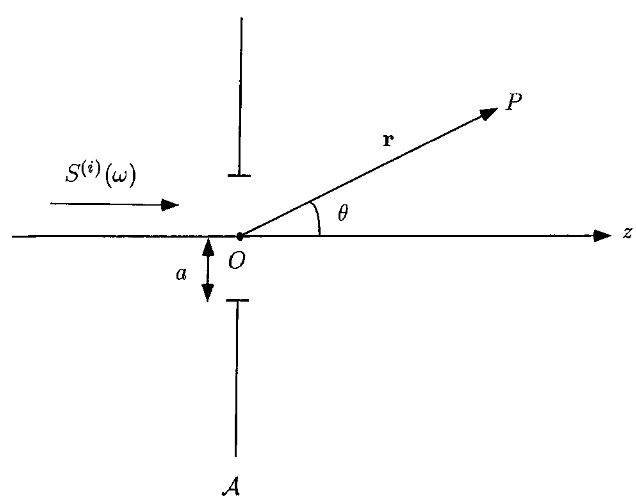

Fig. 1. Illustrating the geometry and notation.

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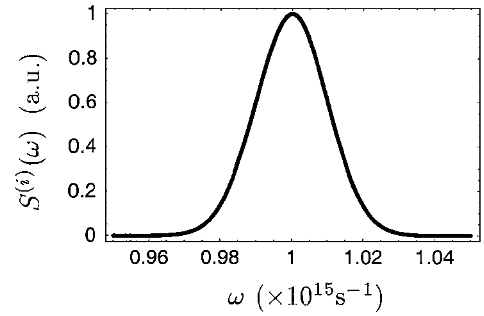

Fig. 2. Gaussian spectrum (in arbitrary units) of the incident wave centered at frequency $\omega_0 = 10^{15}$ s^{-1}. The rms width of the spectral line was taken to be $\sigma = 0.01\omega_0$.

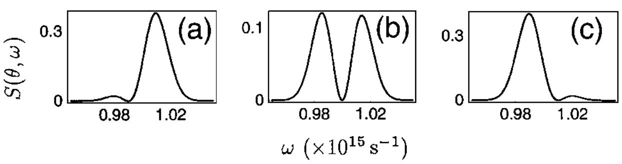

Fig. 3. Spectra (in arbitrary units) of the diffracted light in the far zone in the neighborhood of the first critical direction θ_{c1}: (a) $\theta = 0.99\theta_{c1}$, (b) $\theta = \theta_{c1}$, (c) $\theta = 1.01\theta_{c1}$.

r^{-2}, on propagation in any direction specified by the polar angle θ. Such a self-similarity of the spectrum is, of course, a direct consequence of the fact that in the far zone, the field propagates as an outgoing spherical wave. The evolution of the spectrum of diffracted waves in the far zone in the neighborhood of a critical direction is illustrated schematically in Fig. 4.

Let us also examine the behavior of the spectrum on a small circular loop of radius $\epsilon \ll r$, centered at a particular critical direction. The spectrum of the diffracted light in the far zone is displayed in Fig. 5(b) for different values of the polar angle ϕ, indicated in Fig. 5(a).[6] It is seen from Fig. 5 that in any critical direction $\phi = 0$ ($\theta = \theta_c$) the spectral line is split into two lines, one redshifted, one blueshifted, with the magnitude of the maximum of the line shifted toward the red being slightly greater. There is no such asymmetric splitting in the focusing case.[4] Further, as one moves around the circle clockwise, the spectral line that is shifted toward the blue becomes more and more dominant over the redshifted spectral line, until one crosses the critical direction again, at $\phi = \pi$. At this point, the spectral line has exactly the same profile as the spectral line in the critical direction $\phi = 0$, and from that point on until the complete turn is made, the spectral line that is shifted toward the red is dominant. Such a behavior of the spectrum of diffracted waves in the far zone along a circle crossing a critical direction is qualitatively different from the behavior of the spectrum of focused waves on a circle centered at a singular point, which was studied in Ref. 4.

We note a condition that is necessary for an experimental observation of the spectral anomalies induced by Fraunhofer diffraction. The Airy pattern is formed at a distance $z \gg a^2/\lambda$ from the aperture (Ref. 5, Sec. 8.3.3). A simple estimate with $\lambda \sim 5 \times 10^{-5}$ cm and $a \sim 1$ mm indicates that one must have $z \gg 2$ m.

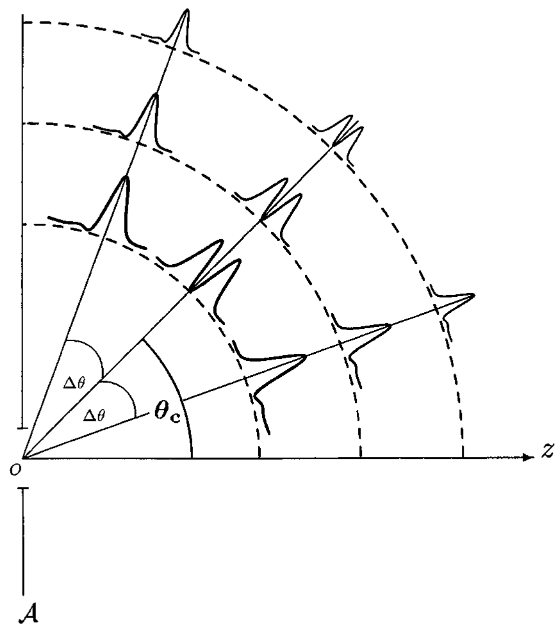

Fig. 4. Schematic illustration of the anomalous behavior of the spectrum in the vicinity of a critical direction θ_c that points toward the first zero of the Airy pattern at frequency ω_0. To illustrate the effect, the angles and distances are not to scale. In particular, the circles shown by the dashed lines are in the far zone of the aperture. The spectrum of the incident light is the same as in Fig. 1.

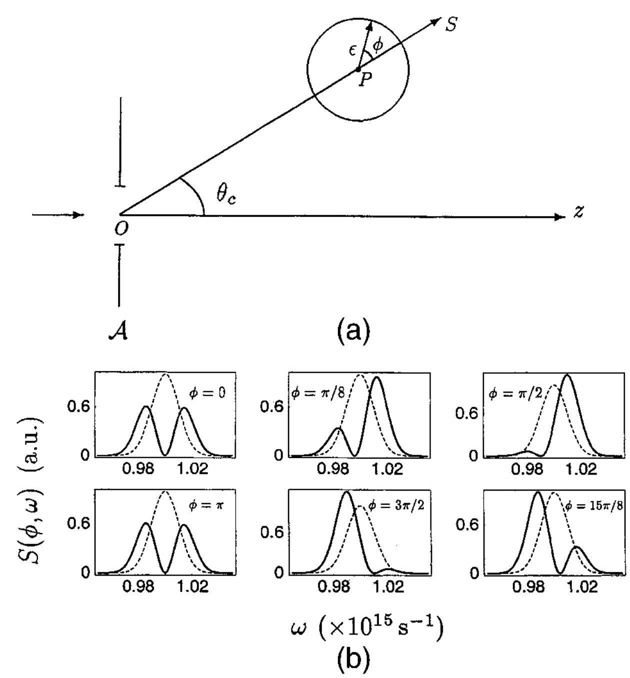

Fig. 5. Spectral changes in the neighborhood of a zero at a point P in the Airy diffraction pattern at frequency ω_0 of a circular aperture, along a small circular loop of radius ϵ centered at P, which is at the distance r from the center O of the aperture. The vector OS points toward the critical direction that makes the angle θ_c with the z axis. The spectrum is displayed for different values of the angle ϕ. The numerical parameters are chosen so that $\epsilon/r\theta_c = 0.005$.

In conclusion, we have shown that the Fraunhofer diffraction of a spatially coherent, polychromatic plane wave at a circular aperture generates remarkable spectral modifications very close to the dark rings in the Airy pattern. Studying the spectral behavior of light with spectral anomalies might be quite rewarding. In particular, the shift of lines toward the blue end or toward the red end of the spectrum might perhaps be used in free-space communication applications. For example, the shift toward one end of the spectrum could be associated with a bit of information such as a "1" say, and the shift in the other direction could be associated with "0". The self-similarity of the far-zone spectrum may make it possible to transmit this information over appreciable distances.

This research was supported by the Defense Advance Research Projects Agency under grant MDA 972011043, by the U.S. Air Force Office of Scientific Research under grant F49260-96-1-0400, and by the Engineering Research Program of the Office of Basic Energy Sciences at the U.S. Department of Energy under grant DE-F02-90 ER 14119. E. Wolf's e-mail address is ewlupus@pas.rochester.edu.

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