

RESEARCH ARTICLE

Spatiotemporal Light Modes: From Elegant Vortices to Vortex-Antivortex Patterns

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ABSTRACT

Spatiotemporal wave packets with nontrivial topologies, such as vortices, have recently attracted sustained interest from the optical community. Identifying the simplest vortex modes is instrumental to design high-dimensional structured light fields and to gain insight into the physics of complex optical dynamics. Herein, we theoretically propose and experimentally demonstrate two families of spatiotemporal light vortices that are exact solutions to the time-dependent wave equation across diverse dispersion regimes. The envelopes of these wave packets, which can be expressed in closed forms, exhibit nontrivial complex field structures and distinct transverse orbital angular momentum patterns. The discovered space-time modes exhibit vortices and antivortices arising from their intrinsic energy density flow that reveals their fundamental characteristics and points to potential applications. The preparation of such intriguing spatiotemporal light states holds significant promise for applications in high-dimensional quantum and classical entanglement, novel photonic topological textures, as well as high-capacity communications and nonlinear optical interactions.

Vortices are topologically nontrivial structures that arise across nature and science, often resulting from spiral energy circulation around a vortex core. In paraxial optics, vortex modes arise as exact solutions to the paraxial wave equation in either free space or a dispersive material medium. The Laguerre-Gaussian (LG) spatial modes, as a representative example of vortex modes, form a complete set of shape-invariant eigenmodes in free space. Following the seminal recognition that spatial LG modes carry longitudinal orbital angular momentum (LOAM) [1], the LG modes have attracted considerable interest and have become central to a wide spectrum of fundamental

phenomena and applications in optical physics [2, 3]. On the other hand, antivortices, which exhibit a saddle-type energy circulation pattern, have been created and observed in both solid-state magnetic materials [4–6] and structured optical fields [7–10]. The antivortices, which typically arise from the interaction of neighboring vortices with opposite topological charges, exhibit topological stability, and thus serve as natural candidates to study topological phenomena [11, 12].

In recent years, spatiotemporal optical vortices (STOVs), featuring tightly coupled spatial and temporal degrees of free-

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dom [13–15] and offering new physical insights into transverse OAM (T-OAM), have attracted rapidly growing interest. Significant progress in exploring STOVs has been made both theoretically [16–18] and experimentally [19–21]. In particular, elementary [18, 22, 23], Bessel-Gaussian [24, 25], and the so-called perfect STOVs with invariant spatiotemporal radius [26–28] have been proposed, realized, and carefully studied in free space and linear dispersive media. Beyond these examples, complex, 3D spatiotemporal structured wave packets have been examined, including STOV combs with ultrafast, time-dependent properties [29] and toroidal light vortices [30, 31], opening new avenues for ultrafast light-matter interactions and high-dimensional optical information processing. Despite this rapid progress, the generation of STOV wave packets remains in its infancy. The vast majority of known STOVs are obtained by solving the time-dependent wave equation for vortex spectra excitation under anomalous dispersion conditions. Consequently, most realizations are limited to standard Laguerre- [19–21] or Bessel-Gaussian [24, 25] spatiotemporal wave packets (STWPs) which exhibit only a simple helical (vortical) phase structure in the space-time plane [18–21]. Identifying complex space-time modes featuring, for instance, vortex or antivortex structures presents an open challenge as the evolution of spatiotemporal light fields is shaped not only by spatial diffraction but also by temporal dispersion.

Here, we aim to advance beyond the currently known forms of STOVs by identifying theoretically and implementing experimentally generalized classes of space-time light states, the STWPs that possess vortex and antivortex mode structure. We derive closed form analytical expressions for the complex field envelopes of the generalized modes and realize the discovered STOVs in the laboratory by precisely sculpting the angular spectrum of an ultrashort laser pulse. In addition, we characterize the modal structure, T-OAM, and energy flow evolution of the generalized STOV classes. In the anomalous dispersion regime, we demonstrate spatiotemporal elegant Laguerre-Gaussian (STELG) wave packets that serve as “elegant” vortex eigenmodes of the medium. Specifically, STELGs possess single donut-shaped intensity profiles and two quantized mode indices associated with radial and azimuthal degrees of freedom. Notably, the STELG eigenmodes exhibit a reduced quality factor, a distinctive feature of iso-propagating vortices. At the same time, we theoretically discover radial-vortex and saddle-vortex STLG wave packets, unveiling nontrivial hyper-phase structures that feature vortex and antivortex energy flows in the normal and anomalous dispersion regimes. These findings are supported by experimental validation and highlight the potential of said generalized space-time modes for high-capacity optical communications [29, 32], nonlinear optics [33, 34], novel structured light field design [13], and the realization of topological quasiparticles of light [35].

Theoretical Model and Experiment

We start by expressing the field envelope of a $(2 + 1)$ D STWP, propagating in a linear dispersive medium, in dimensionless coordinates in terms of its angular spectrum as [26]

$$\Psi(x, t, z) = \int dK \int d\Omega \mathcal{A}(K, \Omega) e^{i(Kx - \Omega\tau)} e^{iz(as\Omega^2 - K^2/a)}. \quad (1)$$

here $s = \text{sgn}(\beta_2)$ and $a = \sqrt{L_x/L_t}$ is a stretching factor quantifying the relative strength of diffraction and dispersion. Further, $L_x = 2\beta_0\sigma_x^2$ and $L_t = 2\sigma_t^2/|\beta_2|$ are characteristic diffraction and dispersion lengths, respectively, given in terms of characteristic transverse spatial σ_x and temporal σ_t scales of the field at the source as well as the propagation constant β_0 and group-velocity dispersion coefficient β_2 . The propagation distance is scaled to $L_d = \sqrt{L_x L_t}$ and τ is a dimensionless retarded time [26].

We can infer at once from Equation (1) that in dimensionless variables, the physics of any envelope evolution in a dispersive medium is completely determined by three factors: the angular spectrum of the source, the type of medium dispersion—normal versus anomalous—and the stretching factor. Moreover, the angular spectrum representation (1) is equivalent to a paraxial wave equation for the slowly varying envelope in the form [26]

$$ia\partial_z\Psi + \partial_{xx}^2\Psi - sa^2\partial_{\tau\tau}^2\Psi = 0. \quad (2)$$

Equation (2) also characterizes the energy density flow of a wave packet propagating in a dispersive medium, which gives rise to angular momentum, as demonstrated in Section S1.

The experimental realization of STWPs of Equation (1) or Equation (2) can be accomplished through complex-amplitude modulation of a pulsed laser beam using a 2D holographic pulse shaper [21]. As illustrated in Figure 1, the spectrum of a femtosecond laser pulse (centered at 1030 nm with a bandwidth of approximately 20 nm) is spatially dispersed along the horizontal direction by the 2D pulse shaper composed of a cylindrical lens, a blazed grating, and a liquid-crystal spatial light modulator (LC-SLM) (Holoeye GAEA-2, 3840×2160 pixels with a pitch of 3.74 μm). The initial pulse has a 1/e spatial width of ~2 mm, and a Fourier-transform-limited duration of ~120 fs (full width at half maximum) and carries positive group-delay dispersion (GDD) of approximately +0.045 ps². The SLM transverse plane corresponds to the angular-spectrum domain, where a phase-only hologram is imposed to achieve complex modulation of the incident wavefront. This hologram encodes both the phase and amplitude components of the target angular-spectrum function, i.e., $\mathcal{A}(K, \Omega)$ [36–38], while the linear chirp term $\exp(iaz\Omega^2)$ is introduced by adding a second-order spectral phase. We provide a more detailed description of the experimentally implemented precise spatial-spectral complex-amplitude modulation using holography in Section S2. We implemented a temporal Fourier transformation by recombining the dispersed spectrum with the cylindrical lens and grating, while spatial diffraction $\exp(-izK^2/a)$ and the Fourier transformation are accomplished through propagation in free space and a spherical lens. The reconstituted pulse, located at the first order (filtered by an iris), is a desired wave packet. To retrieve complex field information of synthesized wave packets, we employed a 3D diagnostic technique to characterize their spatiotemporal morphology. In brief, we introduced a dechirped probe pulse to interfere with the synthesized wave packets, generating a series of time-dependent off-axis spatial interference fringes. By scanning the delay line of the probe pulse, we reconstructed the spatiotemporal profiles of wave packets from the delay-dependent interference patterns [13].

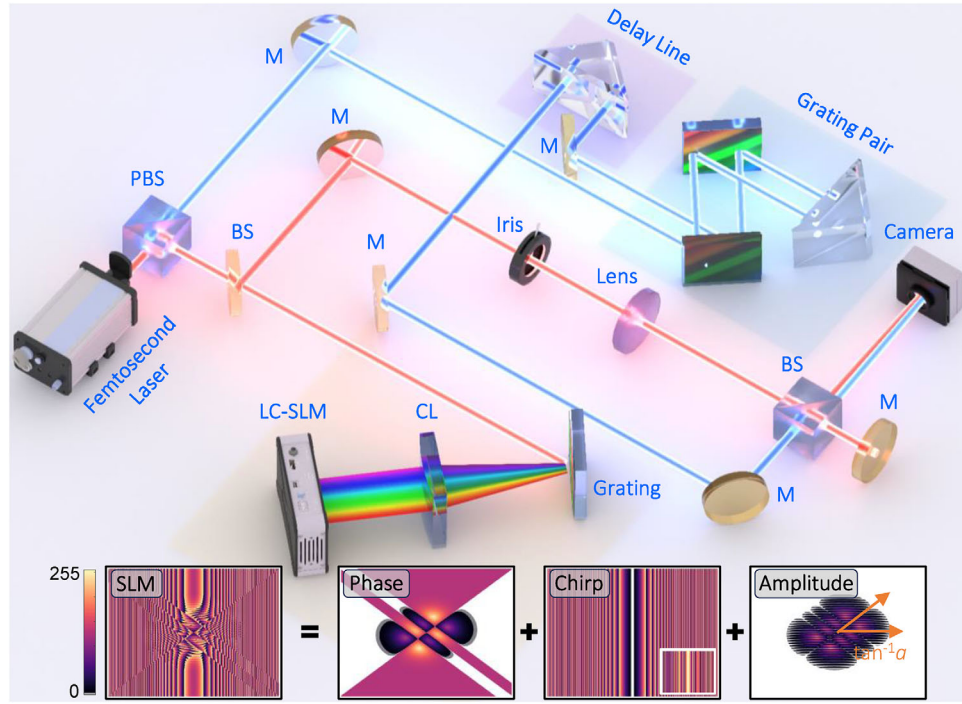


FIGURE 1 | Schematic of experiment setup to generate and characterize light modes in space-time. PBS, polarization beam splitter; BS, beam splitter; CL, Cylindrical lens; LC-SLM, liquid-crystal spatial light modulator; M, Mirror. Bottom: a representative hologram of SLM consisting of the phase, chirp and amplitude terms of the target angular spectrum function of $\mathcal{A}(K, \Omega)$.

Results and Discussion

Anomalous dispersion.—It will prove convenient to transform to stretched polar coordinates which we define as

$$K = a^{1/2}\kappa \cos \theta, \quad \Omega = a^{-1/2}\kappa \sin \theta, \quad (3)$$

and

$$x = a^{-1/2}r \cos \phi, \quad \tau = a^{1/2}r \sin \phi. \quad (4)$$

It follows at once from Equation (4) that

$$r = \sqrt{ax^2 + \tau^2/a}, \quad \phi = \arctan[a^{-1}\tau/x]. \quad (5)$$

In this regime $s = -1$, we express Ψ of any STOV in terms of the angular spectrum as

$$\Psi(r, \phi, z) = \int_0^\infty d\kappa \kappa \int_0^{2\pi} d\theta \mathcal{A}(\kappa, \theta) e^{i\kappa r \cos(\theta+\phi)} e^{-iz\kappa^2}. \quad (6)$$

Next, we consider the following angular spectrum in k -space

$$\mathcal{A}(\kappa, \theta) \propto \kappa^{2p+|l|} e^{-\kappa^2} e^{\mp il\theta}. \quad (7)$$

We drop any immaterial normalization constant hereafter. On substituting from Equation (7) into Equation (6) and performing integration with the aid of the integral representation of the Bessel

function as well as the table integral [39]

$$\int_0^\infty dx x^{2p+|l|+1} e^{-\alpha x^2} J_{|l|}(\beta x) \propto \frac{\beta^{|l|}}{\alpha^{p+|l|+1}} e^{-\frac{\beta^2}{4\alpha}} L_p^{|l|}\left(\frac{\beta^2}{4\alpha}\right), \quad (8)$$

where $L_p^{|l|}(x)$ stands for an associated Laguerre polynomial, l is an integer, and p is a positive integer, we obtain, after elementary algebra, for the field envelope the expression

$$\Psi_{\text{ELG}}(r, \phi, z) \propto \frac{r^{|l|}}{w^{p+|l|+1}} L_p^{|l|}\left(\frac{r^2}{4w}\right) e^{-r^2/4w} e^{\pm il\phi}, \quad (9)$$

where $w = w(z) = 1 + iz$. Equation (9) defines a spatiotemporal elegant Laguerre-Gaussian (STELG) envelope that carries a vortex of topological charge $\pm l$ and radial mode index p .

The STELG wave packets represent a novel class of space-time laser eigenmodes of the wave equation under anomalous dispersion for any z . Although characterized by a radial quantum number indexed by p , the STELGs, unlike previously reported STLG modes [21], exhibit dominant single-ring structures in their transverse space-time planes and spiral energy density flows in the anomalous dispersion regime, as shown in Figure 2. The spatial and spectral scales of the angular spectrum [Equation (7)] used in this regime are set to 0.3 and 13 nm, respectively. The calculated average T-OAM per photon for both the measured and theoretical STELG modes is nearly identical and closely matches its corresponding topological charge. Excellent agreement is observed between the experimental and theoretical results, both revealing spiral

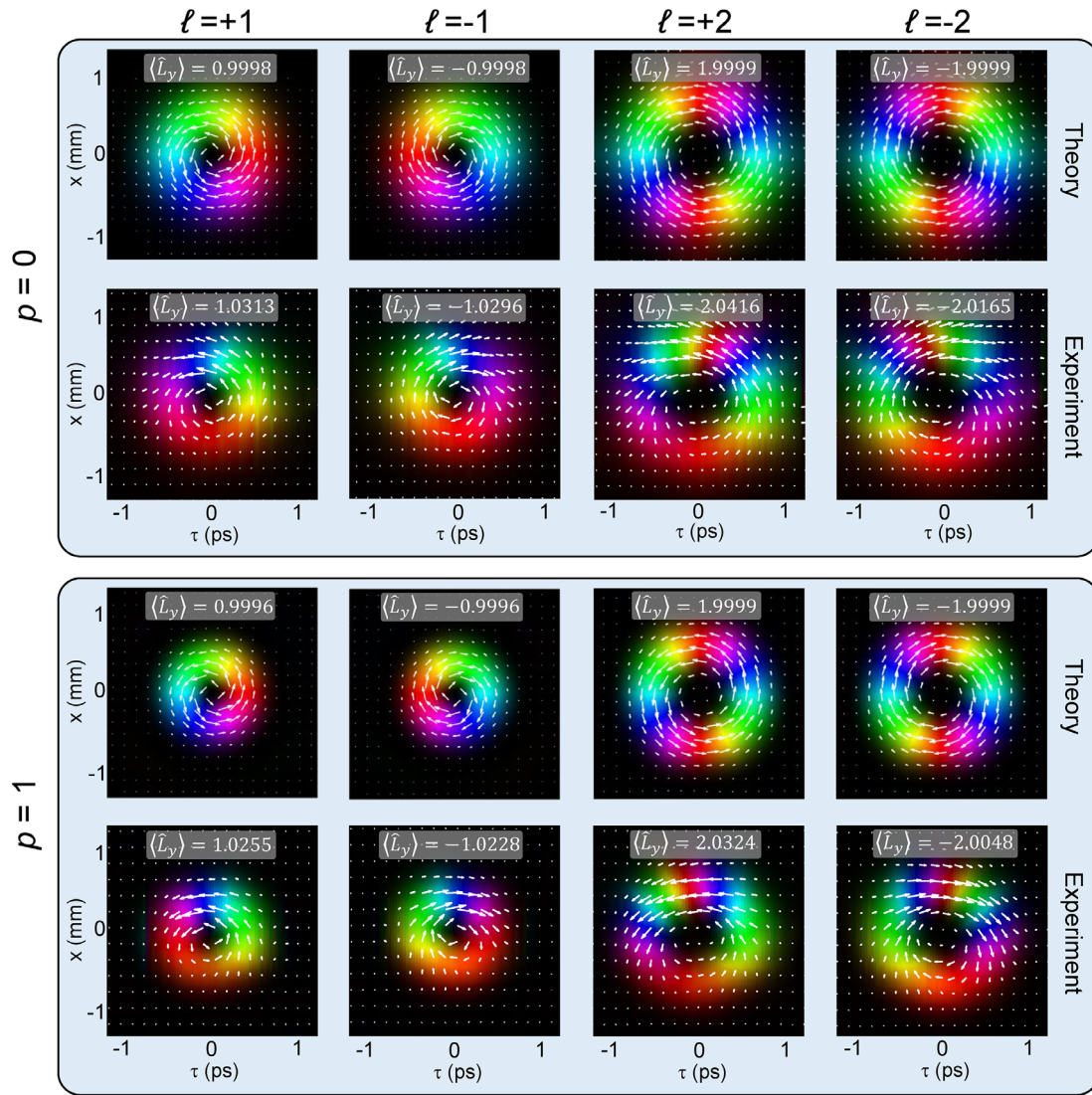


FIGURE 2 | Complex field representation and energy density flow of theoretical [Equation (9)] and measured STELG wave packets for variable l and p in the anomalous dispersion regime ($s = -1$). The bright and hue denote intensity and phase distributions, respectively. The energy density flow is indicated by the white arrows. Other parameters are set to be $a = 1$ and $w = 1$. The average T-OAM values per photon are shown at the top of each panel.

energy flow and an average T-OAM density that corresponds quantitatively to the assigned topological charge. Similar to conventional STLG wave packets [21], STELG wave packets possess intrinsic T-OAM and maintain self-similar propagation profiles in a medium matched to anomalous dispersion (see Section S3).

Further, in optical transmission, the beam size and divergence of vortex beams increase with both their OAM and propagation distance. Although perfect vortex beams have been proposed to mitigate this limitation, their OAM-independent size is maintained only near the focal plane, and their divergence becomes significant for large OAM values [40]. For STELG modes, the energy is confined to the innermost ring, exhibiting a remarkably small beam size and low divergence—both governed by the radial index (Section S4) in the anomalous dispersion regime. Consequently, the proposed STELG eigenmodes show a reduced quality factor M^2 and demonstrate T-OAM inde-

pendent propagation behavior, which is governed by the radial index $p(l) = \text{round}[(|l| + 1)^2/2M^2 - (|l| + 1)/2]$ [41]. This iso-propagation characteristic of STELG wave packets, arising from their being exact solutions to the wave equation, is consistent with the spatial iso-propagation vortices reported in Ref. [42]. Furthermore, owing to their more concentrated energy in a single-ring structure and reduced beam size, the STELG wave packets stand out as promising candidates for future T-OAM-based fiber communication systems [43]. STELG modes also exhibit reduced modal scattering while propagating through complex media [41].

Normal dispersion.—Let us now introduce stretched quadrature variables by the expressions

$$v_{\pm} = \frac{1}{\sqrt{2}} \left(\Omega \sqrt{a} \pm \frac{K}{\sqrt{a}} \right), \quad (10a)$$

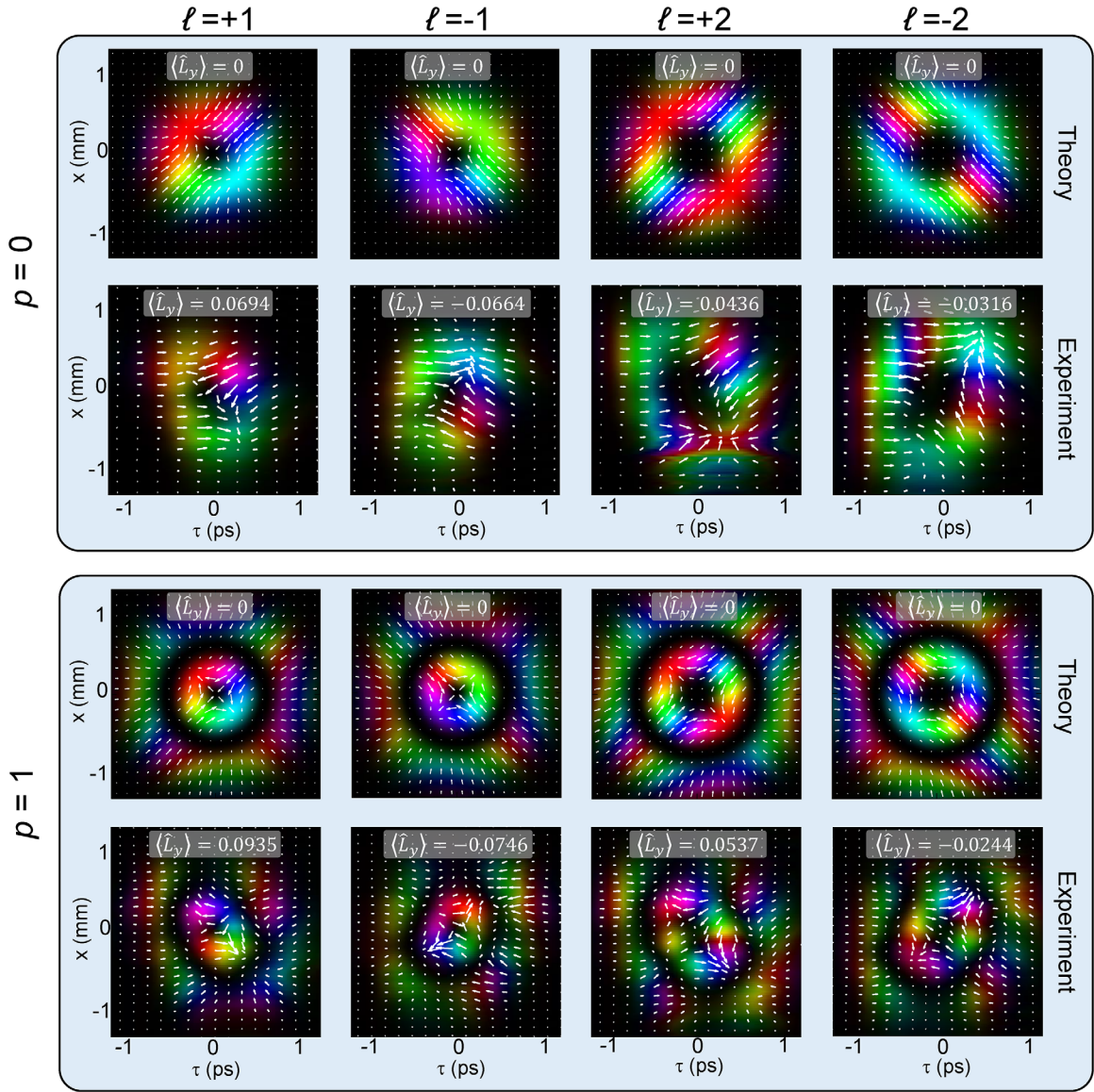


FIGURE 3 | Complex field representation and energy density flow of theoretical [Equation (12)] and measured saddle-radial STLG wave packets for various l and p under normal dispersion of $s = +1$. The bright and hue denote intensity and phase distributions, respectively. The energy density flow is indicated by the white arrows, forms antivortex structures. Other parameters are the same as in Figure 2. The calculated average T-OAM value is close to zero in a medium with normal dispersion across all panels.

$$x_{\pm} = \frac{1}{\sqrt{2}} \left(x \sqrt{a} \pm \frac{\tau}{\sqrt{a}} \right). \quad (10b)$$

Next, we consider the following angular spectrum of the source in the stretched quadrature variables:

$$\mathcal{A}_{p,l}^{\pm}(\nu_+, \nu_-) \propto e^{-(\nu_+^2 + \nu_-^2)} H_{p+|l|}(\sqrt{2}\nu_{\pm}) H_p(\sqrt{2}\nu_{\mp}). \quad (11)$$

here $H_n(x)$ is a Hermite polynomial of order n , l and $p \geq 0$ are integers; the sign $+$ ($-$) corresponds to a positive (negative) sign of the topological charge, $l > 0$ ($l < 0$), of the resulting field as we will see shortly. We can now evaluate the field of any such mode at $z = 1$ in a closed form (Section S5), yielding a family of vortex

carrying modes with the field envelope given by the expression

$$\Psi_{p,l}^{\pm}(x_+, x_-) \propto (x_+ \pm ix_-)^{|l|} L_p^{|l|} \left(\frac{x_+^2 + x_-^2}{4} \right) \times \exp \left(\frac{ix_+ x_-}{4} \right) \exp \left(-\frac{x_+^2 + x_-^2}{8} \right). \quad (12)$$

Alternatively, we can convert Equation (12) to polar coordinates associated with the quadrature variables viz.,

$$x_+ = \tilde{r} \cos \tilde{\phi}; \quad x_- = \tilde{r} \sin \tilde{\phi}, \quad (13)$$

and cast the field envelope of any such mode into the form

$$\Psi_{p,l}^{\pm}(\tilde{r}, \tilde{\phi}) \propto \tilde{r}^{|l|} L_p^{|l|}(\tilde{r}^2/4) e^{-\tilde{r}^2/8} e^{\pm il\tilde{\phi}} e^{i\tilde{r}^2 \sin(2\tilde{\phi})/8}. \quad (14)$$

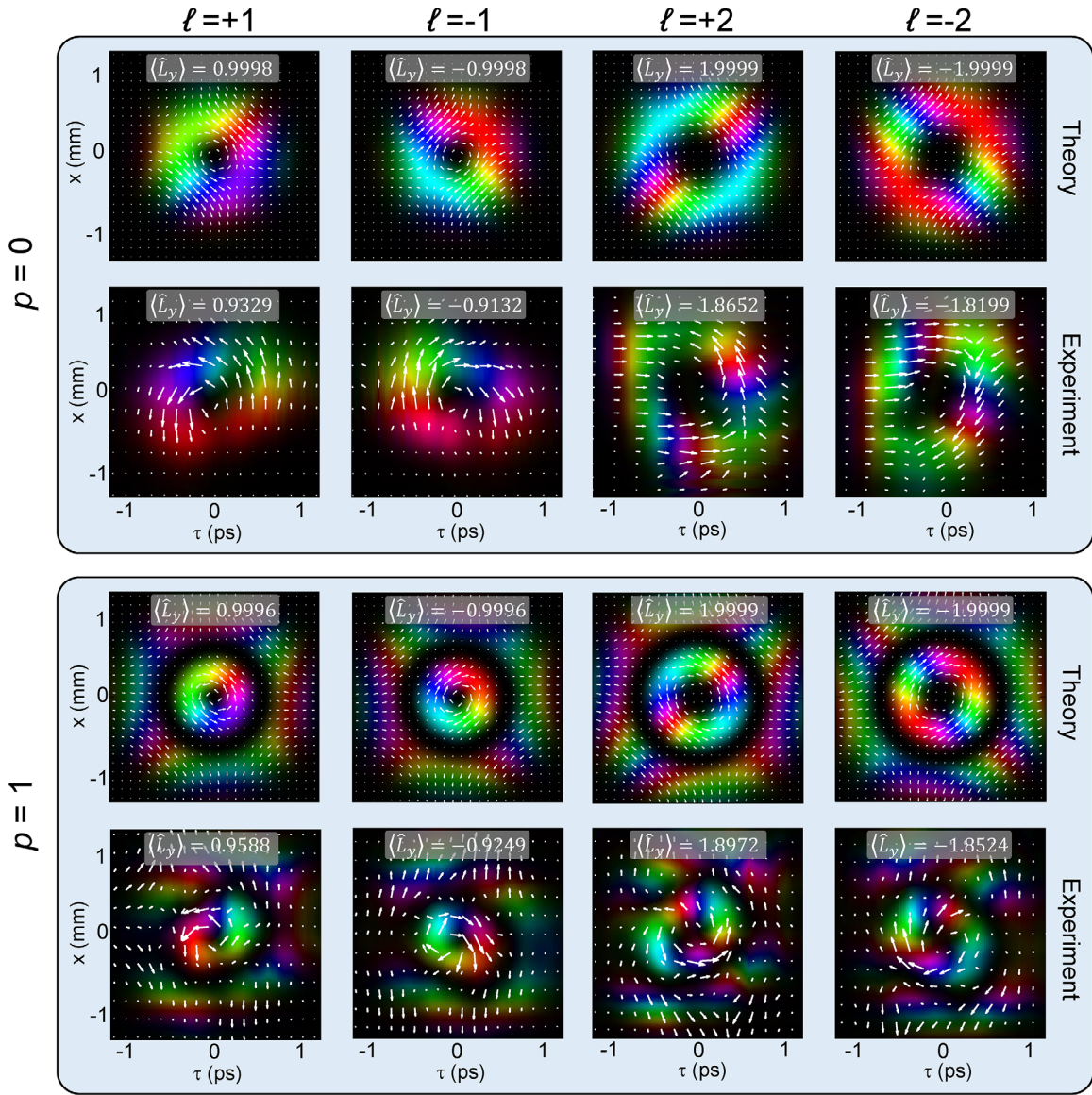


FIGURE 4 | Complex field representation and energy density flow of vortex-saddle STLG wave packets [Equation (12)] for various l and p under anomalous dispersion of $s = -1$. The bright and hue denote intensity and phase distributions, respectively. The energy density flow is indicated by the white arrows, forms a hybrid of vortex and antivortex. Other parameters are the same as in Figure 2. The average T-OAM per photon, obtained from both theoretical and measured results, is calculated at the top.

A quick glance at Equations (12) and (14) reveals that the discovered modes possess dual degrees characteristic of STLG wave packets, namely the azimuthal l and radial p quantum numbers. However, this type of wave packet also carries a distinct spatiotemporal astigmatism, introduced by a hyperbolic phase term $\exp(ix_+x_-/4)$. In both the theoretical analysis and experimental implementation, the spatial and spectral scales of the Hermite–Gaussian angular spectrum [Equation (11)] are set to 0.3 and 13 nm, respectively. In the normal dispersion regime, such an STWP exhibits a hybrid energy density flow with a saddle-radial-like structure (Figure 3), characterized by the presence of central antivortices arising from the surrounding radial energy flow. It is worth noting that similar saddle-like antivortices of low order have indeed been observed in superconducting [44], quantized vortex arrays excited by photonic lattices [45] and ferromagnetic systems [6]. Both theoretical and experimental

results for the average T-OAM density show that this energy-flow configuration corresponds to a net zero average T-OAM per photon.

Remarkably, in the anomalous dispersion regime, the astigmatic in space and time STWP with the angular spectrum of Equation (11) evolves to display a vortex-saddle-like energy flow, as shown in Figure 4, representing hybrid vortex and antivortex characteristics. This hybrid flow emerges from the interplay between the helical phase and the hyperbolic phase in the transverse space-time plane in presence of anomalous dispersion: near the singularity, the spiral energy flow (vortex) dominates, while in the outer regions, the saddle-like energy (antivortex) pattern prevails, giving rise to a vortex-antivortex mode. Although the hyperbolic phase modifies the spatiotemporal wavefront structure of the STWP, it does not alter its average T-OAM value.

Conclusion

In conclusion, we have theoretically introduced two classes of spatiotemporal light modes with nontrivial topological structure, including vortices and antivortices. These STWPs exhibit intensity and phase profiles reminiscent of spatiotemporal Laguerre-Gaussian wave packets, while displaying distinct physical properties in their mode structure, T-OAM, and energy flow compared to conventional STLGs. We have experimentally synthesized novel space-time light modes by meticulously sculpting the angular spectra of pulsed beams at the source. We have successfully created and observed vortex and antivortex patterns within the discovered STWPs. The realization of such vortex-antivortex STWP modes significantly enriches the existing family of vortex modes in the space-time domain, laying the foundation for future advancements in spatiotemporal structured light engineering [13], high-dimensional quantum processing [46, 47], and optical topologies of structured light [35, 48].

In our work, we observe good agreement between our theoretical predictions and our experimental results for the spatiotemporal distributions of intensity, phase, and energy-density flow of the light modes (Section S6). This agreement yields T-OAM values that are consistent with the corresponding theoretical topological charges. The remaining minor deviations from the theory can be attributed to imperfections in the circular symmetry of the intensity distribution and distortions of the spiral wavefront in the spatiotemporal domain. The latter arise from several experimental factors, including optical misalignment, surface curvature, residual modulation of the spatial light modulator, and lens aberrations. Additional discrepancies stem from the non-uniformity of the laser spectrum and mode distortions induced by imperfect matching between spatial diffraction and temporal dispersion.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

All other data are available in the article and its supplementary files or from the corresponding authors upon request.

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Supporting Information

Additional supporting information can be found online in the Supporting Information section.

Supporting File 1: lpor71306-sup-0001-SuppMat.pdf.

Supporting File 2: lpor71306-sup-0002-FigureS1-S9.zip.