Available online at www.sciencedirect.com
SCIENCE DIRECT®
Optics Communications 227 (2003) 73–74
OPTICS COMMUNICATIONS
www.elsevier.com/locate/optcom

The spectral degree of coherence of fully spatially coherent electromagnetic beams

Sergey A. Ponomarenko [a], Emil Wolf [a,b,*]

[a] Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627, USA
[b] The Institute of Optics, University of Rochester, Rochester, New York 14627, USA

Received 30 June 2003; accepted 3 September 2003

Abstract

We determine a general form of the cross-spectral density matrix and the spectral degree of coherence of the electric field of a fully spatially coherent electromagnetic beam. Our result implies that complete spatial coherence does not, in general, impose any conditions on the state of polarization of the beam.

Keyword: Coherence polarization

The general form of the second-order coherence function of a completely spatially coherent scalar field has been obtained many years ago, both in the space-time [1] and in the space-frequency representations [2]. Not long ago results of [2] have indicated the possibility of developing a new technique for determining the phase of a spatially fully coherent field from measurements of the phase of its (unimodular) spectral degree of coherence [3]. First such measurements have already been reported [4,5].

The purpose of this note is to provide a generalization of the result of [2] for statistical electromagnetic beams.

Let us consider a statistically stationary electromagnetic beam propagating close to the z-direction and let $\{E(\mathbf{r},\omega)\}$ be a statistical ensemble which represents the electric field of the beam in the space-frequency domain [6, Section 4.7]. The second-order coherence properties as well as spectral and polarization properties of the electric field of the beam can be characterized by a 2×2 cross-spectral density matrix $\overleftrightarrow{W}$ with the elements

$$W_{ij}(\mathbf{r}_1,\mathbf{r}_2,\omega) = \langle E_i^*(\mathbf{r}_1,\omega)E_j(\mathbf{r}_2,\omega)\rangle. \quad (1)$$

Here $E_i, E_j, (i,j = x,y)$ are Cartesian components of the transverse electric field $\mathbf{E}(\mathbf{r},\omega)$ of the beam. The asterisk in Eq. (1) denotes the complex conjugate and the angle brackets denote the ensemble average.

We wish to determine the most general form of $\overleftrightarrow{W}$ when the beam is fully spatially coherent at frequency ω. In this case, the spectral degree of coherence of the electric field, defined by the expression [7]

* Corresponding author. Fax: +1-716-473-0687.
E-mail address: ewlupus@pas.rochester.edu (E. Wolf).

0030-4018/$ - see front matter © 2003 Published by Elsevier B.V.
doi:10.1016/j.optcom.2003.09.047

$$\eta(\mathbf{r}_1,\mathbf{r}_2,\omega)=\frac{\mathrm{Tr}\overleftrightarrow{W}(\mathbf{r}_1,\mathbf{r}_2,\omega)}{\sqrt{\mathrm{Tr}\overleftrightarrow{W}(\mathbf{r}_1,\mathbf{r}_1,\omega)}\sqrt{\mathrm{Tr}\overleftrightarrow{W}(\mathbf{r}_2,\mathbf{r}_2,\omega)}},\tag{2}$$

is unimodular for all pairs of points $\mathbf{r}_1,\mathbf{r}_2$ within the beam. Consider the obvious inequality

$$\left\langle\left|\sum_{n=1}^{N}a_nE_x(\mathbf{r}_n,\omega)\right|^2\right\rangle\geqslant 0,\tag{3}$$

where N is any positive real number, the a_n's are arbitrary numbers, and $\mathbf{r}_n$'s are position vectors of arbitrary points. It follows from the inequality (3) as well as from the definition of $\overleftrightarrow{W}$ that

$$\sum_{n=1}^{N}\sum_{m=1}^{N}a_m^*a_nW_{xx}(\mathbf{r}_n,\mathbf{r}_m,\omega)\geqslant 0.\tag{4a}$$

Similarly

$$\sum_{n=1}^{N}\sum_{m=1}^{N}b_m^*b_nW_{yy}(\mathbf{r}_n,\mathbf{r}_m,\omega)\geqslant 0,\tag{4b}$$

where b_{m}'s are also arbitrary numbers. On adding these two inequalities, choosing $a_n=b_n$ for all n $(1\leqslant n\leqslant N)$ and recalling the definition (2) of the spectral degree of coherence, we obtain at once the inequality

$$\sum_{n=1}^{N}\sum_{m=1}^{N}a_m^*a_n\eta(\mathbf{r}_n,\mathbf{r}_m,\omega)\geqslant 0.\tag{5}$$

This inequality is identical with an inequality satisfied by the spectral degree of coherence $\mu(\mathbf{r}_1,\mathbf{r}_2,\omega)$ of the scalar theory (see cf. [1,2]). Hence one can draw similar conclusions from inequality (5) regarding η as were previously drawn about μ. In particular, when the beam is completely spatially coherent, one finds that η has necessarily the form

$$\eta(\mathbf{r}_1,\mathbf{r}_2,\omega)=\mathrm{e}^{\mathrm{i}[\phi(\mathbf{r}_1,\omega)-\phi(\mathbf{r}_2,\omega)]},\tag{6}$$

where $\phi(\mathbf{r},\omega)$ is a real function. In this case Eq. (2) implies that

$$\mathrm{Tr}\overleftrightarrow{W}(\mathbf{r}_1,\mathbf{r}_2,\omega)=\mathscr{E}^*(\mathbf{r}_1,\omega)\mathscr{E}(\mathbf{r}_2,\omega),\tag{7}$$

where $\mathscr{E}(\mathbf{r},\omega)=\sqrt{S(\mathbf{r},\omega)}\mathrm{e}^{-\mathrm{i}\phi(\mathbf{r},\omega)}$. Here $S(\mathbf{r},\omega)\equiv\mathrm{Tr}\overleftrightarrow{W}(\mathbf{r},\mathbf{r},\omega)$ is the spectral density of the beam at the point $\mathbf{r}$. The factorization properties (6) and (7) of the spectral degree of coherence and of the trace of the cross-spectral density tensor of the fully spatially coherent beam-like electromagnetic field are the main results of this investigation.

In the scalar case, the factorization condition of the spectral degree of coherence implies the factorization of the cross-spectral density into a product of a function of $\mathbf{r}_1$ and that of $\mathbf{r}_2$. On the other hand, it is seen from Eq. (6) that it is the *trace* of the cross-spectral density matrix, *rather* then every element of it that factorizes. This result has an important physical implication which becomes evident if one recalls that the spectral degree of polarization of a statistical electromagnetic beam depends not only on the diagonal elements of $\overleftrightarrow{W}$ but also on its off-diagonal elements [7]. Since the condition (6) for a spatially completely coherent beam does not involve any off-diagonal elements, it is clear that a fully spatially coherent beam need not be completely polarized. In fact, an example of a fully spatially coherent unpolarized beam is given in [7].

Acknowledgements

This research was supported by the US Air Force Office of Scientific Research under grant F49260-03-1-0138, and by the Engineering Research Program of the Office of Basic Energy Sciences at the US Department of Energy under grant FR-FG02-2ER45992, and by the Defense Advance Research Project Agency under grant MDA 972011043.

References

[1] C.L. Mehta, E. Wolf, A.P. Balachandran, J. Math. Phys. 7 (1966) 133.
[2] L. Mandel, E. Wolf, Opt. Commun. 4 (1981) 247.
[3] E. Wolf, Opt. Lett. 28 (2003) 5.
[4] A. Dogariu, G. Popesku, Phys. Rev. Lett. 89 (2003) 243902.
[5] G. Popesku, L. Delfores, J.C. Vaugham, R.R. Dasari, M.S. Field, Two-dimensional phase reconstruction of polychromatic optical fields, Phys. Rev. Lett., submitted.
[6] L. Mandel, E. Wolf, Optical Coherence and Quantum Optics, Cambridge University Press, Cambridge, 1995.
[7] E. Wolf, Phys. Lett. A 312 (2003) 263, In this reference, the spectral degree of coherence was denoted by $\mu(\mathbf{r}_1,\mathbf{r}_2,\omega)$, but we denote it now by $\eta(\mathbf{r}_1,\mathbf{r}_2,\omega)$ to distinguish it from the corresponding quantity of the scalar theory.