Asymmetric incoherent vector solitons

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We introduce a wide class of partially coherent, partially polarized solitons whose second-order statistical properties include a certain type of reflectional symmetry to be referred to as the *specularity*. We describe generic properties of such solitons in any noninstantaneous saturable nonlinear media. We derive closed form analytical expressions for the intensity profiles as well as the degree of polarization of specular and antispecular partially coherent solitons supported by logarithmically saturable nonlinear media.

The pioneering experiments on the self-trapping of partially coherent light beams in noninstantaneous nonlinear media [1,2] were quickly followed by the development of several mathematically equivalent theories of such partially coherent solitons [3–7]. The analysis of these theories has led to the discovery of certain unique features of partially coherent

spatial solitons [8–11]. To date, partially coherent solitons have been theoretically investigated in a variety of nonlinear media [12].

Thus far, most of the work on partially coherent spatial solitons has been performed in the scalar approximation [13]. Nonetheless, since the original discovery of fully coherent vector solitons in 1974 by Manakov [14], such solitons have been shown to display a variety of interesting properties [12]. At the present time, however, very little is known about partially spatially coherent vector solitons. To our knowledge, the only known two-dimensional [(2+1)D] partially coherent vector solitons, which were found in the media with a thresholding nonlinearity [15], are the ones with axially symmetric intensity profiles of the vector components. This result prompts the intriguing question: Do partially coherent vector solitons possessing other forms of statistical symmetry exist? A related fundamental question is as follows: What is the connection between statistical symmetries of such solitons and those of the light sources generating them? The latter issue could be key to the experimental realization of partially coherent vector solitons with desirable statistical symmetries.

The purpose of the present paper is to address these questions. To this end, we study statistical symmetries of partially coherent, partially polarized spatial solitons in a generic noninstantaneous saturable nonlinear medium, which is homogeneous and isotropic. In particular, we show that, regardless of a functional form of the saturable nonlinearity [16], there exists a class of partially coherent vector solitons whose second-order correlation tensor possesses a certain reflectional symmetry which we refer to as the *specularity*. The components of the correlation tensor of such solitons are either specular or antispecular. We also demonstrate that in the case of a logarithmically saturable medium, closed form analytic expressions can be found for the intensity profiles and the degree of polarization of the specular and antispecular vector solitons. To our knowledge, this is the first ex-

ample of [(2+1)D] partially coherent vector solitons with asymmetric intensity profiles of the vector components [17] and, as a consequence, asymmetric degree of polarization. Another feature of such partially coherent solitons is an explicit dependence of the intensity profiles of their vector components on the position-dependent *twist* phase which vanishes in the fully coherent limit. By comparison, the twist phase of previously discovered partially coherent scalar solitons in logarithmic media can only affect the width of such solitons, but not the shape of their intensity profile [18].

As a starting point, consider a quasimonochromatic, partially spatially coherent source generating a beam of arbitrary polarization or two such uncorrelated sources that generate beams of mutually orthogonal polarizations. For simplicity, we focus on the second case here and assume that the two beams propagate close to the z axis. The electric field of each beam is then of the form: $E_\alpha(\boldsymbol{\rho},z,t') = U_\alpha(\boldsymbol{\rho},z,t')e^{i(k_0 z - \omega_0 t)}$ where U_α is a slowly varying envelope function of the field; $\alpha = x, y$, $k_0 = n_0 \omega_0/c$, ω_0 and n_0 being the carrier frequency and the linear refractive index at that frequency, respectively. Also, $t' = t - z/v_g$ where v_g is the group velocity at ω_0.

Since the two light sources are assumed to be uncorrelated, the second-order statistical properties of the beams generated by such sources are specified by the equal-time correlation tensor $\boldsymbol{\Gamma}$ which has only diagonal elements defined as $\Gamma_{\alpha\alpha}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) = \langle U_\alpha^*(\boldsymbol{\rho}_1,z,t')U_\alpha(\boldsymbol{\rho}_2,z,t')\rangle$, where the angle brackets denote the ensemble averaging. In a noninstantaneous nonlinear medium, each diagonal element of the correlation tensor $\boldsymbol{\Gamma}$ satisfies the nonlinear wave equation

$$i\frac{\partial}{\partial z}\Gamma_{\alpha\alpha}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) + \frac{1}{2k_0}(\nabla_{\perp 1}^2 - \nabla_{\perp 2}^2)\Gamma_{\alpha\alpha}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)$$

$$+ \frac{k_0}{n_0}[n_{NL}(I_1) - n_{NL}(I_2)]\Gamma_{\alpha\alpha}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) = 0, \quad (1)$$

which is a generalization of the similar equation derived in the scalar approximation [5,10]. In Eq. (1) $n_{NL}(I)$ is the nonlinear refractive index, $I_j \equiv I(\boldsymbol{\rho}_j,z) = \Sigma_{\alpha=x,y}\langle|U_\alpha(\boldsymbol{\rho}_j,z)|^2\rangle$, $j = 1,2$, and $\nabla_\perp$ is a gradient transverse to the direction of propagation of the beam. The degree of polarization of such beams, which is defined as the ratio of the intensity of the polarized part of the field to the total intensity, is given by the expression [19]

$$P(\boldsymbol{\rho},z) = \sqrt{1 - \frac{4 \mathrm{Det}\,\boldsymbol{\Gamma}(\boldsymbol{\rho},\boldsymbol{\rho},z)}{\mathrm{Tr}^2 \boldsymbol{\Gamma}(\boldsymbol{\rho},\boldsymbol{\rho},z)}}. \tag{2}$$

The correlation tensor $\boldsymbol{\Gamma}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$ of a partially coherent vector soliton whose diffraction is arrested by nonlinear refraction does not depend on z.

Generalizing the modal theory of partially coherent scalar solitons [4] to the vector case, we can represent the envelope field of each polarization α as a series in the spatial modes:

$$U_\alpha(\boldsymbol{\rho},z,t') = \sum_s a_{s;\alpha}(t') \psi_{s;\alpha}(\boldsymbol{\rho},z). \tag{3}$$

Here $\psi_{s;\alpha}(\boldsymbol{\rho},z)$ is a spatial mode polarized in the direction α, $a_{s;\alpha}(t')$ is a random coefficient of this mode, and the index s labels the modes. The second-order statistical properties of the modes are specified viz.,

$$\langle a_{s;\alpha}^*(t') a_{s;\beta}(t') \rangle = \lambda_{s;\alpha} \delta_{ss'} \delta_{\alpha\beta}, \tag{4}$$

where $\lambda_{s;\alpha} \geq 0$ is the weight of the mode labeled by the index s in the expansion (3). It follows from Eqs. (3) and (4) that the diagonal elements of the correlation tensor can be expressed as

$$\Gamma_{\alpha\alpha}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = \sum_s \lambda_{s;\alpha} \psi_{s;\alpha}^*(\boldsymbol{\rho}_1,z) \psi_{s;\alpha}(\boldsymbol{\rho}_2,z), \tag{5}$$

which provides a link between the correlation tensors of partially coherent vector solitons and the eigenmodes of their common self-induced, via the nonlinearity, waveguide.

Let us consider $[(2+1)\mathrm{D}]$ partially coherent solitons whose correlation functions possess the following symmetry with respect to reflections upon, for instance, the x axis

$$\begin{aligned}\Gamma_\pm(x_1,y_1;x_2,y_2) &= \pm \Gamma_\pm(x_1,-y_1;x_2,y_2) \\ &= \pm \Gamma_\pm(x_1,y_1;x_2,-y_2). \end{aligned} \tag{6}$$

We will refer to the equal-time correlation function possessing such a symmetry as being specular (Γ_+) or antispecular (Γ_-) because it exhibits perfect correlation or anticorrelation at every pair of points Q and $\bar{Q}$ with coordinates (x_1,y_1) and $(x_2,-y_1)$, respectively [20]. It follows that the correlation coefficient of the fields at such pairs of points is $\xi(Q,\bar{Q}) = \Gamma(Q,\bar{Q})/\sqrt{I(Q)I(\bar{Q})} = \pm 1$. It can be inferred from Eqs. (5) and (6) that the intensity of any specular (antispecular) soliton satisfies the condition

$$I_\pm(x,y) = I_\pm(x,-y), \tag{7}$$

and the modes of specular (antispecular) solitons are even (odd) upon reflection with respect to the x axis, i.e.,

$$\psi_{s;\pm}(x,y) = \pm \psi_{s;\pm}(x,-y). \tag{8}$$

In accordance with Eq. (1), the correlation tensor of a partially coherent electromagnetic beam propagating in a nonlinear medium is completely determined by its value in the source plane $z=0$. Further, if the beam is specular or antispecular in the source plane, it can be readily inferred from Eqs. (1) and (7) that the specularity is conserved on propagation in a noninstantaneous nonlinear medium irrespective of the functional form of the nonlinear refractive index. This conservation of specularity implies the possibility of engineering statistical symmetries of partially coherent waves in nonlinear media by controlling the statistical properties of the field in the source plane. In particular, provided the beam is trapped by its self-induced waveguide, one can generate a specular (antispecular) partially coherent soliton by launching specular (antispecular) partially coherent beams into saturable nonlinear media. To demonstrate the possibility of self-trapping of specular and antispecular beams, we consider the case when the total intensity profile of the vector soliton is circularly symmetric, i.e., $I = I(\rho)$ and look for the soliton modes in the form

$$\psi_{nm;x}(\boldsymbol{\rho},z) = u_{nm}(\rho) \cos(m\phi) e^{i\beta_{nm}^2 z/2k_0}, \tag{9}$$

$$\psi_{nm;y}(\boldsymbol{\rho},z) = u_{nm}(\rho) \sin(m\phi) e^{i\beta_{nm}^2 z/2k_0}, \tag{10}$$

where β_{nm} is a propagation constant of the mode labeled by the indices n and m. It is easily seen from Eqs. (8)–(10) that the mode functions $\psi_{nm;x}$ and $\psi_{nm;y}$ are specular and antispecular, respectively. It then follows from Eqs. (1) and (5) that the radial mode function $u_{nm}(\rho)$ satisfies

$$\left\{ \frac{1}{\rho} \frac{d}{d\rho} \left(\rho \frac{d}{d\rho} \right) - \frac{m^2}{\rho^2} + \frac{2k_0^2}{n_0} n_{NL}(I) \right\} u_{nm} = \beta_{nm}^2 u_{nm}, \tag{11}$$

together with the self-consistency condition

$$I(\rho) = \sum_{nm} \lambda_{nm} |u_{nm}(\rho)|^2. \tag{12}$$

Here λ_{nm} is the modal weight distribution which must be the same for both soliton components. Equations (9)–(12) describe vector solitons with specular and antispecular components that can exist in saturable nonlinear media with *any* functional form of the nonlinearity. It is only left to mention that specular or antispecular partially coherent beams can be generated, for example, with the help of a wavefront-folding Michelson interferometer [20]. If a Porro prism of the interferometer changes sign of the y coordinate of the input field, a statistical realization of the output field of such a system is $U(x_1,y_1) \pm U(x_1,-y_1)$ where $U(x,y)$ is the input field and the sign $\pm$ can be adjusted by an appropriate phase shifter. The second-order correlation function of the output beam is then readily seen to be specular or antispecular depending on the sign.

We illustrate these general results by an explicit example of partially coherent vector solitons supported by a noninstantaneous medium with the nonlinear refractive index of the form [21]
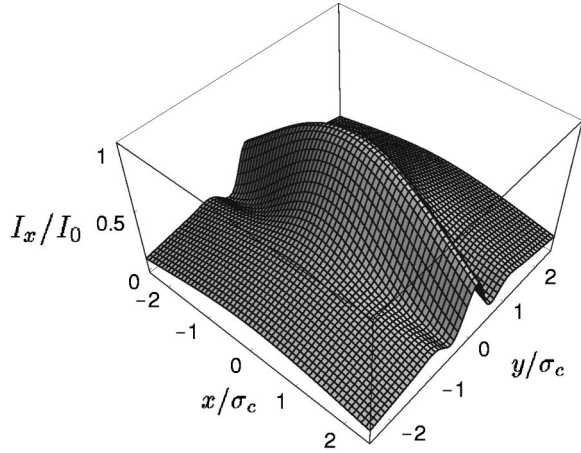
$$n_{NL}(I) = (\Delta n) \ln(I/I_t). \tag{13}$$

FIG. 1. The dimensionless intensity I_x/I_0 of the specular component of an asymmetric incoherent vector soliton as a function of dimensionless coordinates x/σ_c and y/σ_c. The values of the parameters are $u\sigma_c^2=1$, $\sigma_c/\sigma_I=1/2$.

We begin, following Ref. [18], by assuming a circularly symmetric Gaussian shape of the total intensity profile

$$I(\rho)=I_0\exp\left(-\frac{\rho^2}{2\sigma_I^2}\right), \tag{14}$$

where I_0 is the axial intensity, and the constant σ_I represents the soliton width. In this case, the solutions of Eq. (11) have the form

$$u_{nm}(\rho)=(\rho/l_\perp)^{|m|}L_n^{|m|}(\rho^2/l_\perp^2)e^{-\rho^2/2l_\perp^2}, \tag{15}$$

where $L_n^{|m|}(x)$ is an associate Laguerre polynomial of order n ($n=0,1,2,\dots$) with the azimuthal index m ($m=0,\pm 1,\dots$), and $l_\perp$ is a characteristic width of each mode in the transverse plane such that $l_\perp=(k_0^2\Delta n/n_0\sigma_I^2)^{-1/4}$.

We assume that the distribution function of λ_{mn} is the same for both vector components and is given by the expression

$$\lambda_{mn}\propto\frac{n!}{(n+|m|)!}\xi^{n+|m|/2}\eta^m, \tag{16}$$

where ξ and η are non-negative constants related to the soliton parameters. Using the explicit expression for the mode functions and the modal weight distribution and expressing $\cos(m\phi)$ and $\sin(m\phi)$ in terms of $e^{\pm im\phi}$, we can obtain, following the method of Ref. [18], the correlation tensor of specular and antispecular vector solitons in the form

$$\Gamma_\pm(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=\frac{I_0}{2}\exp\left(-\frac{\rho_1^2+\rho_2^2}{4\sigma_I^2}\right)\exp\left[-\frac{(x_1-x_2)^2}{2\sigma_c^2}\right]$$
$$\times\left\{\exp\left[-\frac{(y_1-y_2)^2}{2\sigma_c^2}\right]\cos[u(x_2y_1-x_1y_2)]\right.$$
$$\left.\pm\exp\left[-\frac{(y_1+y_2)^2}{2\sigma_c^2}\right]\cos[u(x_2y_1+x_1y_2)]\right\}. \tag{17}$$

Here Γ_+ and Γ_- stand for Γ_{xx} and Γ_{yy}, respectively.

These solitons are described by three parameters: the soliton width σ_I, its coherence length σ_c, and the twist parameter u, which are related via the self-consistency condition

$$\frac{\sigma_I}{\sigma_c}=\left[\frac{(\alpha^2-1)/2}{1+\sqrt{1+u^2\sigma_c^4(\alpha^2-1)}}\right]^{1/2}, \tag{18}$$

where $\alpha^2=4k_0^2\sigma_I^2\Delta n/n_0$, and $-1\leqslant u\sigma_c^2\leqslant 1$. It follows at once from Eq. (17) that the intensities of specular and antispecular soliton components are

$$I_x(x,y)=\frac{I_t}{2}[1+e^{-2y^2/\sigma_c^2}\cos(2uxy)], \tag{19}$$

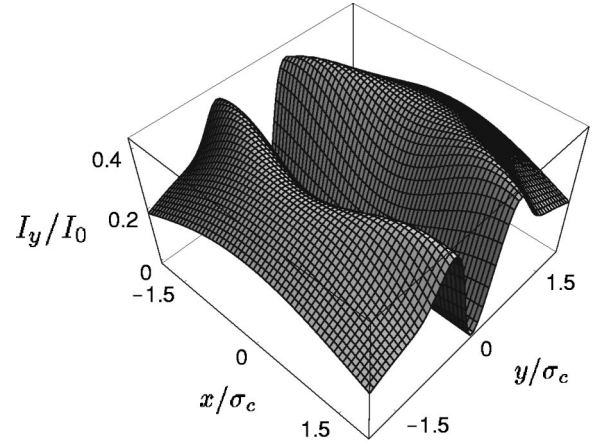$$I_y(x,y)=\frac{I_t}{2}[1-e^{-2y^2/\sigma_c^2}\cos(2uxy)], \tag{20}$$

FIG. 2. The dimensionless intensity I_y/I_0 of the antispecular component of an asymmetric incoherent vector soliton as a function of dimensionless coordinates x/σ_c and y/σ_c. The values of the parameters are $u\sigma_c^2=1$, $\sigma_c/\sigma_I=1/2$.
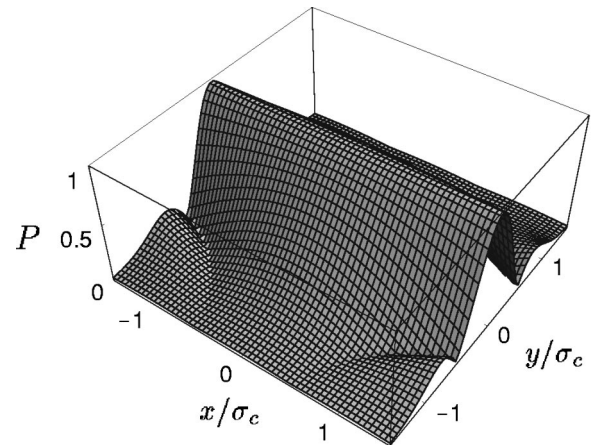

FIG. 3. The degree of polarization of the solitons versus dimensionless coordinates x/σ_c and y/σ_c. The value of the twist parameter u is taken to be such that $u\sigma_c^2=1$.

where $I_t = I_0 \exp[-(x^2+y^2)/2\sigma_I^2]$ is the total intensity of the vector soliton.

It can be readily inferred from Eqs. (19) and (20) that the intensity profiles of the vector-soliton components depend explicitly on the twist parameter u and on the coherence length σ_c, a feature that sets them apart from previously reported twisted scalar solitons [18]. As is manifest from Figs. 1 and 2, this feature of the newly discovered solitons causes breakdown of the axial symmetry of the soliton-component intensity profiles. It is also clear from Eqs. (19) and (20) that the soliton components reproduce the total intensity profile of Eq. (14) so that our solution is self-consistent. On substituting from Eqs. (19) and (20) into Eq. (2), we obtain the degree of polarization of the solitons as

$$P(x,y) = e^{-2y^2/\sigma_c^2}|\cos(2uxy)|. \qquad (21)$$

It is seen from Fig. 3 that the degree of polarization displays an interesting oscillatory behavior due to the influence of the twist parameter. It should also be pointed out that the degree of polarization of the vector solitons is not axially symmetric. Finally, since the total intensity profile of the solitons is single-peaked and circularly symmetric, their stability follows by virtue of the heuristic general criterion $dP/dI_0 > 0$ where $P = \int d^2\rho I$ is the total power of the soliton [22].

In summary, we have discovered a class of partially coherent vector solitons whose vector components are either specular or antispecular. We have described the properties of such solitons in a generic noninstantaneous saturable nonlinear medium. We have also found explicit analytic expressions for the intensities and the degree of polarization of such solitons existing in media with a logarithmic nonlinearity. This work takes what appears to be the first step toward exploring a connection between statistical symmetries of light sources and those of the incoherent solitons generated by such sources in nonlinear media.

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