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Coherence properties of light in Young's interference pattern formed with partially coherent light

Sergey A. Ponomarenko, Emil Wolf *

Department of Physics and Astronomy and Rochester Theory Center for Optical Science and Engineering, University of Rochester, Rochester, NY 14627, USA

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Abstract

It has been shown not long ago that the spectrum of light in Young's interference pattern formed by partially coherent light differs, in general, from the spectrum of light incident on the pinholes; and, moreover, that the spectrum depends on the position of the point of observation. In this paper we extend the analysis by deriving expressions for the spectral degree of coherence and for the cross-spectral density of intensity fluctuations of the light at any pair of points in the interference pattern. We illustrate the general results by examples. © 1999 Elsevier Science B.V. All rights reserved.

1. Introduction

Although it has been about 200 years ago since Thomas Young made a major contribution to the wave theory of light by describing a now famous two-beam interference experiment, somewhat similar experiments still attract a good deal of attention; both in classical and in quantum optics. In the domain of classical optics it was discovered not long ago that if the pinholes are illuminated with broadband light, in which case no interference fringes will be formed, light in the region of superposition of the two beams emerging from the pinholes nevertheless contains important physical information. This is revealed when the light is analyzed spectrally [1–4]. One then finds that the spectrum of the light in the region of superposition depends not only on the spectrum of the light incident on the pinholes but

also on the degree of coherence of the light as well as on the position of the observation point. Using this result one can deduce the value of the spectral degree of coherence of the light at the two pinholes from measurements of the observed spectral changes. In turn, the knowledge of the spectral degree of coherence may provide, in some cases, information about the source of radiation. A possibility of using this effect for remote sensing, for example, by the use of van Cittern–Zernike theorem has been studied and an application to determination of the angular separation of double stars has also been considered [5–7].

Somewhat analogous, though more restricted experiments have been performed with matter waves, especially with neutrons [8,9] (see also [10]). In the domain of quantum optics, Young's interference experiments carried out in recent years have provided experimental tests of such basic concepts of quantum physics as indistinguishability and photon localization ([11], Section 12.11).

* Corresponding author.

It is clear from these remarks that interference experiments of Young's type are still of considerable scientific interest, in spite of the long time which has gone by since Thomas Young first described them. In this paper we discuss an aspect of Young's interference experiment which does not appear to have been previously considered, namely the change in the state of coherence of the light in the region of superposition, when light incident on the pinholes is partially coherent.

2. Coherence properties of Young's interference pattern formed with partially coherent light

Suppose that two pinholes P_1 and P_2 in a plane opaque screen \mathcal{A} are illuminated by partially coherent light (Fig. 1). We will derive expressions for the spectrum, the cross-spectral density and the spectral degree of coherence of the light in the region of superposition of the two beams emerging from the pinholes.

We represent the field at any point P by an ensemble of space frequency realizations $\{U(P, \omega)\}$ ([11], Section 4.7). The field at any pair of points Q_1 and Q_2 in the interference pattern in the plane \mathcal{B} of observation, taken to be parallel to the plane \mathcal{A} ,

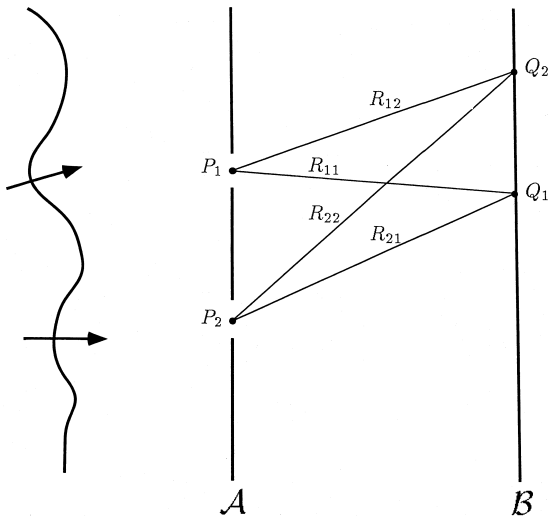


Fig. 1. Illustrating the notation relating to the calculation of the cross-spectral density $W(Q_1, Q_2, \omega)$ in the Young's interference pattern formed with partially coherent light.

expressed in terms of the realizations of the field at the pinholes, is then given by the formulae

$$U(Q_1, \omega) = \frac{ikA}{2\pi} \left[U(P_1, \omega) \frac{e^{ikR_{11}}}{R_{11}} + U(P_2, \omega) \frac{e^{ikR_{21}}}{R_{21}} \right], \quad (1a)$$

$$U(Q_2, \omega) = \frac{ikA}{2\pi} \left[U(P_1, \omega) \frac{e^{ikR_{12}}}{R_{12}} + U(P_2, \omega) \frac{e^{ikR_{22}}}{R_{22}} \right]. \quad (1b)$$

Here R_{ij} is the distance from the pinhole P_i to the point Q_j ($i, j = 1, 2$), in the plane \mathcal{B} , A is the area of each pinhole and $k = \omega/c$, c being the speed of light in vacuum. We have assumed that the angles of incidence and diffraction are small. The cross-spectral density of the light at Q_1 and Q_2 is given by the expression ([11], Eq. (4.7-37))

$$W(Q_1, Q_2, \omega) = \langle U^*(Q_1, \omega) U(Q_2, \omega) \rangle, \quad (2)$$

where the angular brackets denote ensemble average and asterisk denotes the complex conjugate. On substituting from Eqs. (1a) and (1b) into Eq. (2) one finds that

$$W(Q_1, Q_2, \omega) = \left(\frac{kA}{2\pi} \right)^2 \left[W(P_1, P_1, \omega) \frac{e^{ik(R_{12}-R_{11})}}{R_{12}R_{11}} + W(P_2, P_2, \omega) \frac{e^{ik(R_{22}-R_{21})}}{R_{22}R_{21}} + W(P_1, P_2, \omega) \frac{e^{ik(R_{22}-R_{11})}}{R_{22}R_{11}} + W(P_2, P_1, \omega) \frac{e^{ik(R_{12}-R_{21})}}{R_{12}R_{21}} \right], \quad (3)$$

where $W(P_1, P_2, \omega) = \langle U^*(P_1, \omega)U(P_2, \omega) \rangle$ is the cross-spectral density of the light at the pinholes P_1 and P_2 . When $P_1 = P_2$ we have

$$\begin{aligned} W(P_1, P_1, \omega) &= \langle U^*(P_1, \omega)U(P_1, \omega) \rangle \\ &= S(P_1, \omega), \end{aligned} \quad (4a)$$

$$\begin{aligned} W(P_2, P_2, \omega) &= \langle U^*(P_2, \omega)U(P_2, \omega) \rangle \\ &= S(P_2, \omega), \end{aligned} \quad (4b)$$

representing the spectral densities of the light at the points P_1 and P_2 , respectively. It will be useful to introduce the spectral degree of coherence of the light at the two pinholes ([11], Section 4.3):

$$\mu(P_1, P_2, \omega) = \frac{W(P_1, P_2, \omega)}{\sqrt{S(P_1, \omega)}\sqrt{S(P_2, \omega)}}. \quad (5)$$

Using the definitions (4) and (5) the formula (3) can be rewritten as

$$\begin{aligned} W(Q_1, Q_2, \omega) &= \left(\frac{kA}{2\pi} \right)^2 \left\{ K_{11}^* K_{12} S(P_1, \omega) \right. \\ &\quad + K_{21}^* K_{22} S(P_2, \omega) \\ &\quad + 2\sqrt{S(P_1, \omega)S(P_2, \omega)} \\ &\quad \times [K_{11}^* K_{22} \mu(P_1, P_2, \omega) \\ &\quad \left. + K_{12} K_{21}^* \mu(P_2, P_1, \omega)] \right\}, \end{aligned} \quad (6)$$

where

$$K_{ij} = \frac{e^{ikR_{ij}}}{R_{ij}}. \quad (7)$$

Usually the spectra at P_1 and P_2 are approximately the same, in which case we will write

$$S(P_1, \omega) \approx S(P_2, \omega) = S(P, \omega). \quad (8)$$

Also, it follows from the definition of the spectral degree of coherence that

$$\mu(P_1, P_2, \omega) = \mu^*(P_2, P_1, \omega). \quad (9)$$

Using Eqs. (8) and (9) the formula (6) may be rewritten as

$$\begin{aligned} W(Q_1, Q_2, \omega) &\approx \left(\frac{kA}{2\pi} \right)^2 S(\omega) [K_{11}^* K_{12} + K_{21}^* K_{22} \\ &\quad + K_{11}^* K_{22} \mu(P_1, P_2, \omega) \\ &\quad + K_{12} K_{21}^* \mu^*(P_1, P_2, \omega)]. \end{aligned} \quad (10)$$

It then follows at once from (10) that the spectrum of the light at a point Q_0 in the region of superposition is given by the expression

$$\begin{aligned} S(Q_0, \omega) &\equiv W(Q_0, Q_0, \omega) = S^{(1)}(\omega)S^{(2)}(\omega) \\ &\quad + 2\sqrt{S^{(1)}(\omega)S^{(2)}(\omega)} |\mu(P_1, P_2, \omega)| \\ &\quad \times \cos[\beta + k(R_{20} - R_{10})], \end{aligned} \quad (11)$$

where R_{10} and R_{20} denote the distances from the pinholes P_1 and P_2 to the point Q_0 and β is the phase of the spectral degree of coherence $\mu(P_1, P_2, \omega)$ of the light at the two pinholes (see Fig. 1). Further

$$S^{(1)}(\omega) = \left(\frac{kA}{2\pi R_{10}} \right)^2 S(\omega), \quad (12)$$

represents the spectral density which would be observed at the point Q_0 if only the pinhole P_1 were open, the other pinhole being closed. Similarly,

$$S^{(2)}(\omega) = \left(\frac{kA}{2\pi R_{20}} \right)^2 S(\omega), \quad (13)$$

represents the spectral density which would be observed at the point Q_0 if only the pinhole P_2 were open. In most cases of interest $S^{(2)}(\omega) \approx S^{(1)}(\omega)$, and Eq. (11) then reduces to

$$\begin{aligned} S(Q_0, \omega) &\approx 2S^{(1)}(\omega) \{1 + |\mu(P_1, P_2, \omega)| \\ &\quad \times \cos[\beta + k(R_{20} - R_{10})]\}. \end{aligned} \quad (14)$$

This formula, which is in agreement with a result derived in [1], shows that the spectrum of the light at any point Q_0 in the plane \mathcal{B} of observation differs, in general, from the spectrum of the light incident on the pinholes; and that, moreover, it depends on the location of the point Q_0 and on the spectral degree of coherence of the light at the pinholes. In terms of $W(Q_1, Q_2, \omega)$ and $S(Q, \omega)$ the spectral degree of coherence of the light in the plane \mathcal{B} of observation can be calculated from the formula

$$\mu(Q_1, Q_2, \omega) = \frac{W(Q_1, Q_2, \omega)}{\sqrt{S(Q_1, \omega)}\sqrt{S(Q_2, \omega)}} \quad (15)$$

where $W(Q_1, Q_2, \omega)$ and $S(Q, \omega)$ are given by the expressions (10) and (11) respectively. In Section 4 we will illustrate some of the formulas derived in the present section by examples.

3. Correlation properties of intensity fluctuations in Young's interference pattern

It is also of interest to examine the intensity correlations across the interference pattern formed in Young's interference experiment with partially coherent light. We will now consider this problem, restricting ourselves to the situation when the light obeys Gaussian statistics. This will be the case, for example, when the light is of thermal origin. Unlike in the case considered in the previous section, it is now convenient to use the space-time rather than the space-frequency representation.

Let $V(P, t)$ be the complex analytic signal representation ([11], p. 92) of the field, assumed to be a wide sense stationary random process of zero mean, at a point P at time t and let

$$I(Q, t) = V^*(Q, t)V(Q, t) \quad (16)$$

be the instantaneous intensity at the point Q . Further let

$$\Gamma_I(Q_1, Q_2, \tau) = \langle I(Q_1, t)I(Q_2, t + \tau) \rangle \quad (17)$$

be the intensity correlation function. On substituting from (16) into (17) one obtains for Γ_I the following expression in terms of the complex field variable $V(Q, t)$:

$$\begin{aligned} \Gamma_I(Q_1, Q_2, \tau) &= \langle V^*(Q_1, t)V(Q_1, t) \\ &\quad \times V^*(Q_2, t + \tau)V(Q_2, t + \tau) \rangle. \end{aligned} \quad (18)$$

Since the field is assumed to obey Gaussian statistics, one can express the fourth-order correlation function on the r.h.s of Eq. (18) in terms of the second-order correlations using the moment theorem for Gaussians random processes ([12], Appendix I) and one finds that

$$\begin{aligned} \Gamma_I(Q_1, Q_2, \tau) &= \langle U^*(Q_1, t)U(Q_1, t) \rangle \\ &\quad \times \langle U^*(Q_2, t + \tau)U(Q_2, t + \tau) \rangle \\ &\quad + \langle U^*(Q_1, t)U(Q_2, t + \tau) \rangle \\ &\quad \times \langle U^*(Q_2, t + \tau)U(Q_1, t) \rangle \\ &= \langle I(Q_1, t) \rangle \langle I(Q_2, t) \rangle \\ &\quad + \Gamma_V(Q_1, Q_2, \tau)\Gamma_V^*(Q_1, Q_2, \tau), \end{aligned} \quad (19)$$

where

$$\Gamma_V(Q_1, Q_2, \tau) = \langle V^*(Q_1, t)V(Q_2, t + \tau) \rangle \quad (20)$$

is the well-known mutual coherence function of the field. In deriving Eq. (19) we have made use of the assumed statistical stationarity of the field V .

In many cases it is more convenient to deal with the correlations of the intensity fluctuations

$$\Delta I = I(Q, t) - \langle I(Q, t) \rangle \quad (21)$$

rather than with the correlations of the intensity itself. Evidently

$$\begin{aligned} \Gamma_{\Delta I}(Q_1, Q_2, \tau) &\equiv \langle \Delta I(Q_1, t)\Delta I(Q_2, t + \tau) \rangle \\ &= \Gamma_I(Q_1, Q_2, \tau) - \langle I(Q_1, t) \rangle \\ &\quad \times \langle I(Q_2, t + \tau) \rangle. \end{aligned} \quad (22)$$

It follows from Eqs. (20), (22) and (19) that

$$\Gamma_{\Delta I}(Q_1, Q_2, \tau) = |\Gamma_V(Q_1, Q_2, \tau)|^2. \quad (23)$$

Using this result one may readily obtain an expression for the cross-spectral density function $W_{\Delta I}(Q_1, Q_2, \omega)$ of the intensity fluctuations. According to the generalized Wiener–Khinchin theorem ([11], p. 427) it is given by the temporal Fourier transform of the space-time correlation function $\Gamma_{\Delta I}$. On taking the Fourier transform on both sides of (23) and using the convolution theorem for Fourier transforms one finds at once that

$$\begin{aligned} W_{\Delta I}(Q_1, Q_2, \omega) &= \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} W_V^*(Q_1, Q_2, \omega') \\ &\quad \times W_V(Q_1, Q_2, \omega + \omega'). \end{aligned} \quad (24)$$

Here we have used the fact that the cross-spectral density function $W_V(Q_1, Q_2, \omega)$ is the Fourier transform of the space-time correlation function $\Gamma_V(Q_1, Q_2, \tau)$, given by (20). As is known from coherence theory in the space-frequency domain, W_V is the same function as the one defined formally in a somewhat different manner by Eq. (2) ([11], see 4.7, especially Eqs. (4.7-2) and (4.7-8)).

The formula (24) is a generalization of an expression derived by Mandel [13] many years ago for the case when the points Q_1 and Q_2 coincide.

4. Some implications of the theory and examples

In [1] the changes in the spectrum of light in the interference pattern were studied for the situation where the pinholes were illuminated by a distant, spatially incoherent source. We will consider the same situation, but instead of studying spectral changes we will examine the changes in the spectral degree of coherence and in the cross-spectral density of field and of the intensity fluctuations of the light in the region of superposition.

As in [1] we consider the situation where the two pinholes P_1 and P_2 are illuminated by a uniform, spatially incoherent, secondary, circular source σ of radius a . We assume the arrangement to be symmetric, with the plane \mathcal{A} of the pinholes being parallel to the source plane σ and with both the midpoint between the pinholes and the center of the circular source located on the normal to the plane of the pinholes. We assume that the pinholes are in the far zone of the source. As is well known, the light incident upon the pinholes will be partially coherent. Its spectral degree of coherence $\mu(P_1, P_2, \omega)$ may be determined by the use of one of the reciprocity relations for the far field generated by a planar, secondary, quasi-homogeneous source [14]. The relation is analogous to the far-zone form of the van Cittern–Zernike theorem ([15], Section 10.4.2), and one finds that

$$\mu(P_1, P_2, \omega) = \frac{2J_1(\omega\alpha d/c)}{(\omega\alpha d/c)}. \quad (25)$$

Here J_1 is the Bessel function of the first kind and the first order, d is the distance between the two pinholes, α is the angle which the radius of the circular source subtends at the midpoint between the two pinholes, c being the speed of light in vacuum (see Fig. 2). On substituting from (25) into (10) and expressing all the K_{ij} 's in terms of d , α and R one obtains for the cross-spectral density $W(Q_1, Q_2, \omega)$ the expression

$$\begin{aligned} W(Q_1, Q_2, \omega) &= \frac{2(\omega/c)^2 A^2}{(2\pi R)^2} S(\omega) e^{i(\omega/c)(x_1^2 - x_2^2)/2R} \\ &\times \left\{ \cos[\omega d(x_1 - x_2)/2cR] + \mu(P_1, P_2, \omega) \right. \\ &\left. \times \cos[\omega d(x_1 + x_2)/2cR] \right\}. \quad (26) \end{aligned}$$

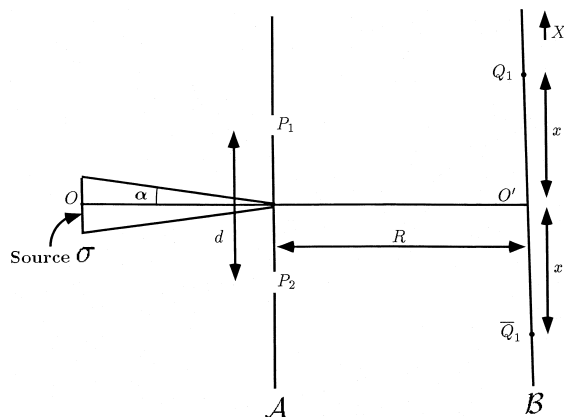


Fig. 2. Illustrating the notation relating to the calculation of the spectral degree of coherence at the pinholes P_1 and P_2 illuminated by light from spatially incoherent, uniform, planar, secondary, circular source σ of radius a . The plane of observation \mathcal{B} is in the far zone of the source.

Here $\mu(P_1, P_2, \omega)$ is the spectral degree of coherence of the source at two points P_1 and P_2 . For a spatially incoherent, planar, secondary source which we will consider in our numerical examples, it is given by (25). In deriving (26) we have assumed, for simplicity, that the observation points Q_1 and Q_2 lie in \mathcal{B} on the line parallel to the line joining the pinholes (see Fig. 2), but our analysis can readily be extended to the case of arbitrary locations of the observation points.

Suppose now that the two points Q_1 and Q_2 in the observation plane \mathcal{B} are located symmetrically with respect to the line OO' (see Fig. 2). We will refer to this line as the optical axis. Let us denote by \bar{Q}_1 the point located symmetrically with respect to Q_1 and let $x = x_1 = -x_2$. It then follows from (26) that

$$\begin{aligned} W(Q_1, \bar{Q}_1, \omega) &= \frac{2(\omega/c)^2 A^2}{(2\pi R)^2} S(\omega) \{ \cos[\omega xd/cR] \\ &+ \mu(P_1, P_2, \omega) \}. \quad (27) \end{aligned}$$

Next, we substitute from (27) into (15) and obtain the following expression for the spectral degree of

coherence at a pair of points located symmetrically with respect to the optical axis of the system:

$$\mu(Q_1, \bar{Q}_1, \omega) = \frac{\cos\{\omega xd/cR\} + \mu(P_1, P_2, \omega)}{1 + \mu(P_1, P_2, \omega)\cos\{\omega xd/cR\}}. \quad (28)$$

We note that when $|\cos(\omega xd/Rc)| = 1$, $|\mu(Q_1, \bar{Q}_1, \omega)| = 1$. This result implies that at pairs of points

in the observation plane \mathcal{B} , located symmetrically with respect to the optical axis, for which

$$\frac{\omega xd}{Rc} = \pi n, \quad (n = 0, \pm 1, \pm 2, \dots), \quad (29)$$

the field will be spatially completely coherent at frequency ω , regardless of the spectral degree of coherence of the source. Further, we note that the

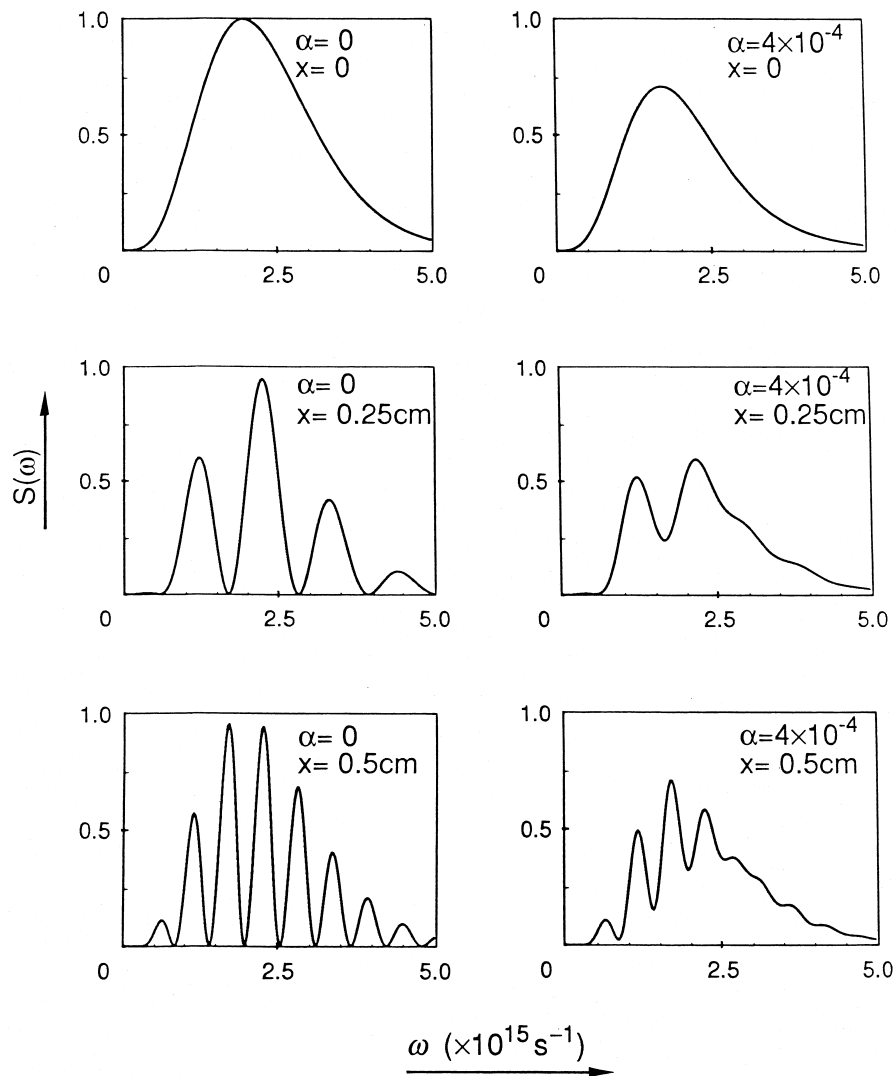


Fig. 3. The spectra at different points Q located at off-axis distance x in the observation plane \mathcal{B} , when the angular radius of the source subtended at the pinholes is α (in radians). The spectrum of the incident light is assumed to be blackbody spectrum at temperature 3000 K. The parameters used in the calculation are $d = 0.1 \text{ cm}$ and $R = 150 \text{ cm}$. (After D.F.V. James and E. Wolf [1]).

pairs of points given by (29) correspond to symmetrically located maxima of intensity (at frequency ω) ([14], p. 261). Hence the absolute value of the spectral degree of coherence reaches its maximum at pairs of maxima of intensity located symmetrically with respect to the optical axis. On the other hand, at pairs of symmetrically located minima of intensity in the observation plane \mathcal{B} , $\cos(\omega xd/Rc) = 0$, i.e., at points for which

$$\frac{\omega xd}{Rc} = \pi [n + (1/2)], \quad (n = 0, \pm 1, \pm 2, \dots). \quad (30)$$

Eq. (28) implies that $\mu(Q_1, \bar{Q}_1, \omega) = \mu(P_1, P_2, \omega)$. We see from (28) that at the points given by (30) the absolute value of the spectral degree of coherence reaches its minimum. It follows that the modulus of the spectral degree of coherence of the field at any pair of points symmetrically located with respect to

the optical axis in the plane of observation is bounded from below by the value of the modulus of the spectral degree of coherence of the source.

We will now illustrate the change in the state of coherence of light in the plane of observation by a numerical example. We consider Young's interference pattern formed with partially coherent thermal light generated by a planar secondary source with the spectral degree of coherence given by (25). We assume that the spectrum of light incident on the pinholes is given by Plank's law:

$$S(\omega) = \frac{B\omega^3}{\exp(\hbar\omega/k_B T) - 1}. \quad (31)$$

In this formula, \hbar and k_B are Plank's and Boltzmann's constants respectively, T is the absolute temperature of the source, and B is a positive constant. The spectrum calculated from (11) is shown in Fig. 3. In Fig. 4 the spectral degree of coherence of the

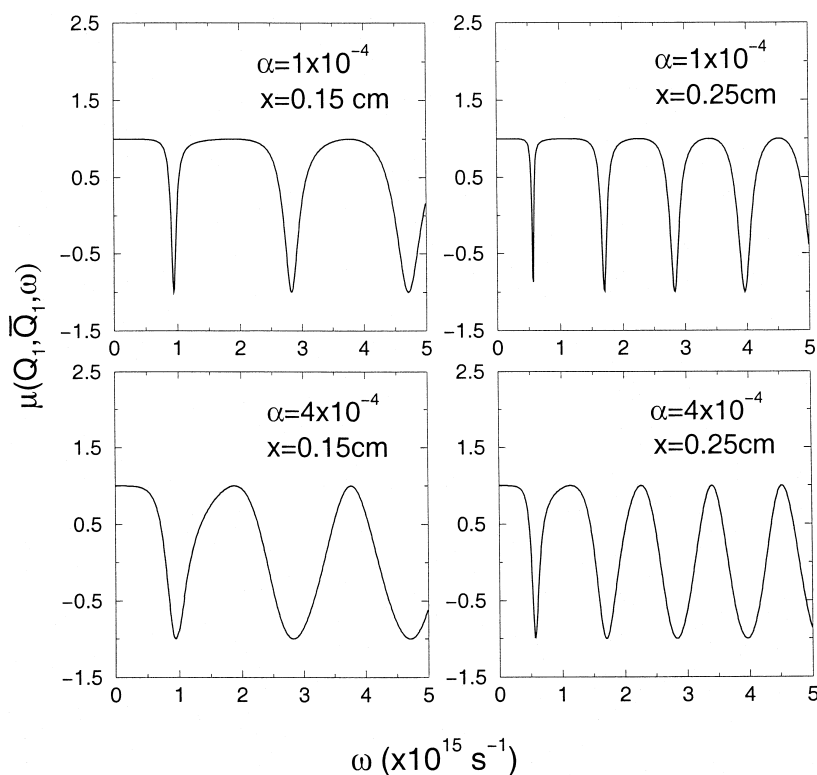


Fig. 4. The spectral degree of coherence of the field at symmetrically located points in the observation plane \mathcal{B} , separated by the distance $2x$. The angular radius of the source subtended at the pinholes is α (in radians). The spectrum of the incident light is given by Plank's formula with the temperature $T = 3000$ K. The calculations were performed with the same choice of parameters d and R as in [1], i.e., $d = 0.1$ cm and $R = 150$ cm.

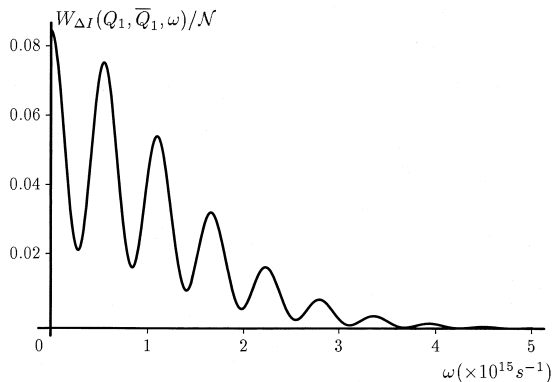


Fig. 5. The normalized cross-spectral density function of the intensity fluctuations of the light at symmetrically located points in the observation plane \mathcal{B} , separated by the distance $2x = 1$ cm. The normalization constant is $\mathcal{N} = B^2 A^4 / 4(\pi R c)^4$. The source subtends the angle $\alpha = 3 \times 10^{-4}$ rad at the pinholes. The spectrum of the incident light as well as the other numerical parameters are the same as those in Figs. 3 and 4.

field at a pair of points located symmetrically with respect to the optical axis in the plane of observation, calculated from (28) is plotted against frequency, for different values of the parameters. On comparing Figs. 3 and 4 we see that, apart from the limiting case $\alpha \rightarrow 0$, (which corresponds to a completely coherent source), the spectrum as well as the spectral degree of coherence of the field in the region of superposition exhibits rapid oscillations as a function of frequency. These oscillations are caused by interference of any particular frequency component of the light emerging from the pinholes. In Fig. 5 the normalized cross-spectral density of the intensity fluctuations is plotted against the frequency ω . We see that it decreases as the frequency increases, but it also oscillates, with approximately the same period as the cross-spectral density, calculated for the same separation between the points Q_1 and Q_2 .

In summary, we have derived general expressions for the cross-spectral density and the spectral degree of coherence of the light, and the cross-spectral density of the intensity fluctuations of light obeying Gaussian statistics at any pair of points in the region of superposition of two partially coherent beams. We found that regardless of the spectral degree of coherence of the source, the field across the pattern is

spatially completely coherent at all pairs of the maxima of intensity located symmetrically with respect to the optical axis. We also noted that at the minima of the intensity at pairs of points located symmetrically with respect to the optical axis, the absolute value of the spectral degree of coherence attains its minimum, which is equal to that of the spectral degree of coherence of the source at the same frequency. Finally, we have also briefly discussed the coherence properties of the Young's pattern formed with partially coherent light, radiated by a uniform, spatially incoherent, planar, secondary source.

Acknowledgements

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