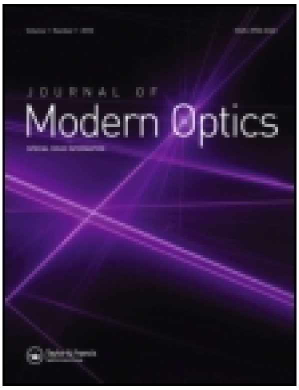

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Change in the polarization of partially coherent electromagnetic beams propagating through the turbulent atmosphere

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We study the changes in the degree of polarization of an electromagnetic Gaussian Schell-model beam, as the beam propagates through the turbulent atmosphere. We demonstrate that, within the framework of the Tatarskii model of the turbulent atmosphere, the degree of polarization of the beam changes appreciably at relatively short propagation distances in the atmosphere. In the long-propagation distance limit, however, we find that the degree of polarization of the beam tends to the value that it has in the source plane.

1. Introduction

In the last two decades, there has been substantial interest in studying the propagation of partially coherent beams in the turbulent atmosphere (see, for example, [1–8]). However, these studies have been carried out in the scalar approximation. It has been previously claimed [9] that the polarization properties of electromagnetic beams generated by fully spatially coherent sources do not appreciably change on propagation in the turbulent atmosphere. To our knowledge, the issue of turbulence-induced changes in the degree of polarization of partially coherent beams has, however, not been previously addressed. Recently a unified theory of coherence and polarization has been formulated [10]. This theory makes it possible to study the changes in the degree of polarization of beams propagating in any linear medium, deterministic or random [11]. With the help of this theory, we examine the changes in the spectral degree of polarization of a wide class of electromagnetic beams, the so-called Gaussian Schell-model (GSM) beams, propagating through the turbulent atmosphere, under the condition that the smaller of a typical width of the beam and of the transverse coherence length of the light in the source plane is much smaller than the inner scale of turbulence. In this approximation the problem becomes analytically tractable. Our results may find applications in long-range communications as well as in long-range interferometry with partially coherent light.

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2. Propagation of degree of polarization in the turbulent atmosphere

Consider a partially coherent electromagnetic beam propagating close to the z axis in the turbulent atmosphere. Let $\{\boldsymbol{E}(\boldsymbol{r},\omega)\}$ be a statistical ensemble of the fluctuating component of frequency ω of the electric field of the beam at a point P($\boldsymbol{r}$). The coherence properties of the beam can then be characterized by the 2×2 (electric) cross-spectral density matrix [12]

$$\mathbf{W} \equiv W_{ij}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \left\langle E_i^*(\mathbf{r}_1, \omega) E_j(\mathbf{r}_2, \omega) \right\rangle \qquad (i, j = x, y), \tag{1}$$

where x and y are two mutually orthogonal directions perpendicular to the beam axis and asterisk denotes the complex conjugate. Each member of the statistical ensemble $\boldsymbol{E}(\boldsymbol{r},\omega)$ at any point P, specified by the position vector $\boldsymbol{r} = (\boldsymbol{\rho}, z > 0)$ (figure 1), can be determined from the knowledge of the field $\boldsymbol{E}^{(0)}(\boldsymbol{\rho}', \omega)$ in the source plane $z = 0$ by using the extended Huygens–Fresnel principle (see p.113, equation (68), of [13])

$$\boldsymbol{E}(\boldsymbol{\rho}, z; \omega) = -\frac{\mathrm{i}k \exp(\mathrm{i}kz)}{2\pi z} \iint \boldsymbol{E}^{(0)}(\boldsymbol{\rho}'; \omega) \exp\left(\mathrm{i}k \frac{(\boldsymbol{\rho} - \boldsymbol{\rho}')^2}{2z}\right) \exp[(\psi(\boldsymbol{\rho}, \boldsymbol{\rho}', z; \omega)] \, \mathrm{d}^2 \rho'. \tag{2}$$

Here $\boldsymbol{\rho}'$ and $\boldsymbol{\rho}$ are the transverse vectors specifying points in the planes $z = 0$ and $z =$ constant > 0 respectively, ψ denotes a random factor representing the effect of turbulence on the propagation of a spherical wave, and $k = \omega/\mathrm{c}$. On substituting from equation (2) into equation (1), we obtain for the cross-spectral density matrix of the electric field in the plane $z =$ constant > 0 the expression

$$\begin{aligned} W_{ij}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z; \omega) = \left(\frac{k}{2\pi z}\right)^2 \iint \mathrm{d}^2 \boldsymbol{\rho}_1' \iint \mathrm{d}^2 \boldsymbol{\rho}_1' \, W_{ij}^{(0)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2'; \omega) \\ \times \exp\left[-\mathrm{i}k \frac{(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_1')^2 - (\boldsymbol{\rho}_2 - \boldsymbol{\rho}_2')^2}{2z}\right] \\ \times \left\langle \exp(\psi^*(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1', z, \omega) + \psi^*(\boldsymbol{\rho}_2, \boldsymbol{\rho}_2', z, \omega)) \right\rangle_m, \end{aligned} \tag{3}$$

where $\langle .... \rangle_m$ denotes averaging over the ensemble of statistical realizations of the turbulent medium.

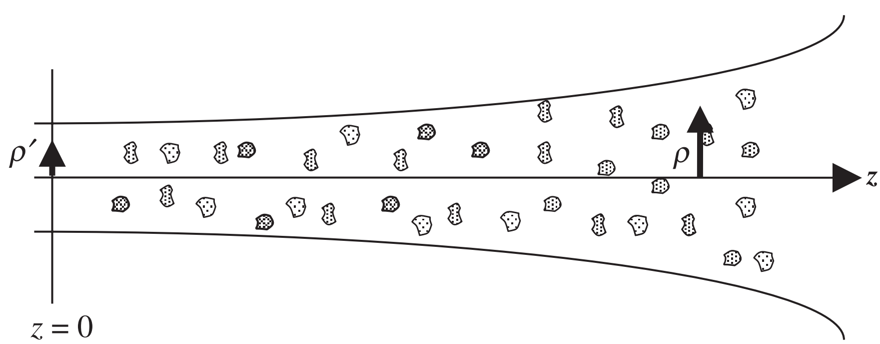

Figure 1. Illustration of the notation.

The spectral degree of polarization $\mathcal{P}$ of an electromagnetic beam is given by the expression [10]

$$\mathcal{P}(\boldsymbol{\rho},\, z;\, \omega) = \left(1 - \frac{4\ \det[\mathbf{W}(\boldsymbol{\rho}, \boldsymbol{\rho}, z;\, \omega)]}{\{\mathrm{Tr}[\mathbf{W}(\boldsymbol{\rho}, \boldsymbol{\rho}, z;\, \omega)]\}^2}\right)^{1/2}. \tag{4}$$

In order to calculate the spectral degree of polarization in a plane $z = \text{constant} > 0$ we first take $\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2 = \boldsymbol{\rho}$ in equation (3), which then reduces to

$$W_{ij}(\boldsymbol{\rho}, z;\, \omega) = \left(\frac{k}{2\pi z}\right)^2 \iint \mathrm{d}^2\boldsymbol{\rho}'_1 \iint \mathrm{d}^2\boldsymbol{\rho}'_2\; W^{(0)}_{ij}(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2;\, \omega) \exp\left(-\mathrm{i}k \frac{(\boldsymbol{\rho} - \boldsymbol{\rho}'_1)^2 - (\boldsymbol{\rho} - \boldsymbol{\rho}'_2)^2}{2z}\right) \times \left\langle \exp[\psi^*(\boldsymbol{\rho}, \boldsymbol{\rho}'_1, z;\, \omega) + \psi(\boldsymbol{\rho}, \boldsymbol{\rho}'_2, z;\, \omega)]\right\rangle_m. \tag{5}$$

The last term in the integrand of the right-hand side of equation (5) can be shown to be given by the expression (see section 12.2.3 of [13]) (see [14])

$$\left\langle \exp[\psi^*(\boldsymbol{\rho}, \boldsymbol{\rho}'_1, z;\, \omega) + \psi(\boldsymbol{\rho}, \boldsymbol{\rho}'_2, z;\, \omega)]\right\rangle_m = \exp\left(-4\pi^2 k^2 z \int_0^1 \int_0^\infty \kappa \Phi_n(\kappa)[1 - J_0(\kappa\xi|\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2|)]\, \mathrm{d}\kappa\, \mathrm{d}\xi\right), \tag{6}$$

where the function Φ_n is the spatial power spectrum of the refractive-index fluctuations of the turbulent atmosphere and J_0 is the Bessel function of the first kind and zero order. It can be shown (see appendix B of [7]) that, under the strong fluctuation condition of turbulence and provided that the spectral coherence length of the source is much shorter than the inner scale of turbulence, the right-hand side of equation (6) may be approximated by the expression

$$\left\langle \exp[\psi^*(\boldsymbol{\rho}, \boldsymbol{\rho}'_1, z;\, \omega) + \psi(\boldsymbol{\rho}, \boldsymbol{\rho}'_2, z;\, \omega)]\right\rangle_m \approx \exp\left(-(1/3)\pi^2 k^2 z |\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2|^2 \int_0^\infty \kappa^3 \Phi_n(\kappa)\, \mathrm{d}\kappa\right). \tag{7}$$

3. Examples

To illustrate the behaviour of the spectral degree of polarization of partially coherent electromagnetic beams in turbulent media, we shall apply the theory to a particular case, namely to an electromagnetic GSM beam propagating through the atmosphere. We assume, for simplicity, that the off-diagonal elements of the electric cross-spectral density matrix of the beam in the source plane have zero value (i.e. that $W^{(0)}_{xy}(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, z, \omega) \equiv W^{(0)}_{yx}(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, z, \omega) = 0$), where $\boldsymbol{\rho}'_1$ and $\boldsymbol{\rho}'_2$ are two-dimensional position vectors of two points in the source plane $z = 0$. The electric cross-spectral density matrix of such a source can be expressed in the form

$$\boldsymbol{W}^{(0)}(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega) \equiv \begin{bmatrix} \left[S^{(0)}_x(\boldsymbol{\rho}'_1, \omega) S^{(0)}_x(\boldsymbol{\rho}'_2, \omega)\right]^{1/2} \eta^{(0)}_{xx}(\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2, \omega) & 0 \\ 0 & \left[S^{(0)}_y(\boldsymbol{\rho}'_1, \omega) S^{(0)}_y(\boldsymbol{\rho}'_2, \omega)\right]^{1/2} \eta^{(0)}_{yy}(\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2, \omega) \end{bmatrix}. \tag{8a}$$

Here $S_j^{(0)}$ are the spectral intensity distributions and $\eta_{jj}^{(0)}$ are the spectral correlation coefficients in the source plane, defined as

$$\eta_{ij}(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega) = \frac{\left\langle E_i^*(\boldsymbol{\rho}'_1, \omega) E_j(\boldsymbol{\rho}'_2, \omega)\right\rangle}{\left[\left\langle E_i^*(\boldsymbol{\rho}'_1, \omega) E_i(\boldsymbol{\rho}'_1, \omega)\right\rangle\right]^{1/2}\left[\left\langle E_j^*(\boldsymbol{\rho}'_2, \omega) E_i(\boldsymbol{\rho}'_2, \omega)\right\rangle\right]^{1/2}} \tag{8b}$$

are given by expression

$$S_j^{(0)}(\boldsymbol{\rho}', \omega) = I_j \exp\left(\frac{-\boldsymbol{\rho}'^2}{2\sigma^2}\right), \qquad (j = x, y), \tag{9}$$

$$\eta_{jj}^{(0)}(\boldsymbol{\rho}'_2 - \boldsymbol{\rho}'_1, \omega) = \exp\left(-\frac{(\boldsymbol{\rho}'_2 - \boldsymbol{\rho}'_1)^2}{2\delta_{jj}^2}\right), \qquad (j = x, y), \tag{10a}$$

$$\eta_{jk}^{(0)}(\boldsymbol{\rho}'_2 - \boldsymbol{\rho}'_1, \omega) = 0, \qquad (j \neq k). \tag{10b}$$

The parameters I_j, σ and δ_{jj} depend on the frequency ω, in general. It follows at once from equations (4) and (8) that the spectral degree of polarization of such a source is given by the formula

$$\mathcal{P}(\boldsymbol{\rho}, \omega) = \frac{\left|I_x - I_y\right|}{I_x + I_y}. \tag{11}$$

On substituting from equations (8)–(10) into equation (5) and using equation (7), we obtain for the elements of the electromagnetic cross-spectral density matrix of the beam (with $\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2 = \boldsymbol{\rho}$) in the plane $z = $ constant > 0 the expressions

$$W_{xx}(\boldsymbol{\rho}, \boldsymbol{\rho}, z; \omega) = \frac{I_x}{\Delta_x^2(z)} \exp\left(-\frac{\boldsymbol{\rho}^2}{2\sigma^2 \Delta_x^2(z)}\right), \tag{12a}$$

$$W_{yy}(\boldsymbol{\rho}, \boldsymbol{\rho}, z; \omega) = \frac{I_y}{\Delta_y^2(z)} \exp\left(-\frac{\boldsymbol{\rho}^2}{2\sigma^2 \Delta_y^2(z)}\right), \tag{12b}$$

where

$$\Delta_j^2 = \left[1 + \frac{z^2}{(k\sigma)^2}\left(\frac{1}{4\sigma^2} + \frac{1}{\delta_{jj}^2}\right) + \frac{z^3}{\sigma^2}\left(\frac{2}{3}\pi^2 \int_0^\infty \kappa^3 \Phi_n(\kappa)\, \mathrm{d}\kappa\right)\right], \qquad (j - x, y). \tag{13}$$

In the Tatarskii model of atmospheric turbulence (see section (3.3.2) of [13]), the spectrum of the refractive-index fluctuations is given by the expression

$$\Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-11/3} \exp\left(-\frac{\kappa^2}{\kappa_m^2}\right), \tag{14}$$

where C_n^2 is the structure parameter of the refractive index and $\kappa_m = 5.92/l_0$, l_0 being the inner scale of turbulence. Substituting from equation (14) into equation (13) and performing the necessary integration yields

$$\Delta_j^2(z) = \left[1 + \frac{z^2}{(k\sigma)^2}\left(\frac{1}{4\sigma^2} + \frac{1}{\delta_{jj}^2}\right) + \frac{z^3}{\sigma^2}(1.093 C_n^2 l_0^{-1/3})\right]. \tag{15}$$

On substituting from equation (12) into equation (4), we find that the spectral degree of polarization of the GSM beam propagating in the turbulent atmosphere is given by the expression

$$\mathcal{P}(\boldsymbol{\rho}, z; \omega) = \frac{\left|(I_x/\Delta_x^2)\exp(-\boldsymbol{\rho}^2/(2\sigma^2\Delta_x^2)) - (I_y/\Delta_y^2)\exp(-\boldsymbol{\rho}^2/(2\sigma^2\Delta_y^2))\right|}{(I_x/\Delta_x^2)\ \exp(-\boldsymbol{\rho}^2/(2\sigma^2\Delta_x^2)) + (I_y/\Delta_y^2)\ \exp(-\boldsymbol{\rho}^2/(2\sigma^2\Delta_y^2))}. \qquad (16)$$

A detailed analysis of equation (16) reveals that the behaviour of the spectral degree of polarization of such a beam is strongly affected by turbulence. However, it follows from equations (15) and (16) that, in the long-distance limit, the spectral degree of polarization of the GSM beam approaches its initial value, irrespective of the magnitude of the spatial correlation lengths of the electric field components in the source plane. On the other hand, the long-distance asymptotic value of the spectral degree of polarization of such a beam propagating in free space, ($C_n^2 = 0$), can readily be shown to be given by the expression

$$\mathcal{P}_\infty = \frac{|I_x\delta_{xx}^2 - I_y\delta_{yy}^2|}{I_x\delta_{xx}^2 + I_y\delta_{yy}^2}, \qquad (17)$$

provided that $\sigma \gg \max(\delta_{xx}, \delta_{yy})$. It is seen from equation (17) that, if a beam is initially unpolarized and if, for example, $\delta_{xx} \gg \delta_{yy}$, the beam becomes fully polarized after propagating over a sufficiently long distance in *free space*. This result is in agreement with a previous study of the correlation-induced polarization changes of beams propagating in free space [15]. Under the same conditions, however, the degree of polarization of the beam in turbulent media approaches zero in the long-distance limit. It also follows at once from equation (17) that if the intensities of the x and y components of the field in the source plane are such that $I_y \gg I_x$ and $\delta_{xx} \gg \delta_{yy}$, and if also $I_y/I_x \approx \delta_{xx}^2/\delta_{yy}^2$, then a highly polarized beam becomes almost completely unpolarized on propagation over a sufficiently long distance in *free space*. On the other hand, the spectral degree of polarization of such a beam, which can decrease on propagation in the turbulent atmosphere over relatively short distances, tends to unity as the propagation distance of the beam increases.

In figure 2, we compare the on-axis behaviours of the spectral degrees of polarization of the GSM beam propagating in the turbulent atmosphere and in free space, as functions of z for different values of the spectral degree of polarization of the field in the source plane. The figure illustrates quantitatively the relative roles of turbulence and diffraction on the polarization properties of the beam. It is also seen that, as the beam propagates over a sufficiently long distance through the turbulent atmosphere, its spectral degree of polarization tends to the same value that it has in the source plane.

To summarize, we have studied the changes in the spectral degree of polarization of a class of partially coherent electromagnetic beams propagating in turbulent media. We have found that the behaviours of the spectral degrees of polarization of such beams depend strongly on their value in the source plane. Somewhat surprisingly, we have found that, in the long-distance limit, the beams do not become unpolarized, as one might perhaps expect; rather their degree of polarization approaches the value that they have in the source plane. This result holds regardless of the state of coherence of the light in the source plane.

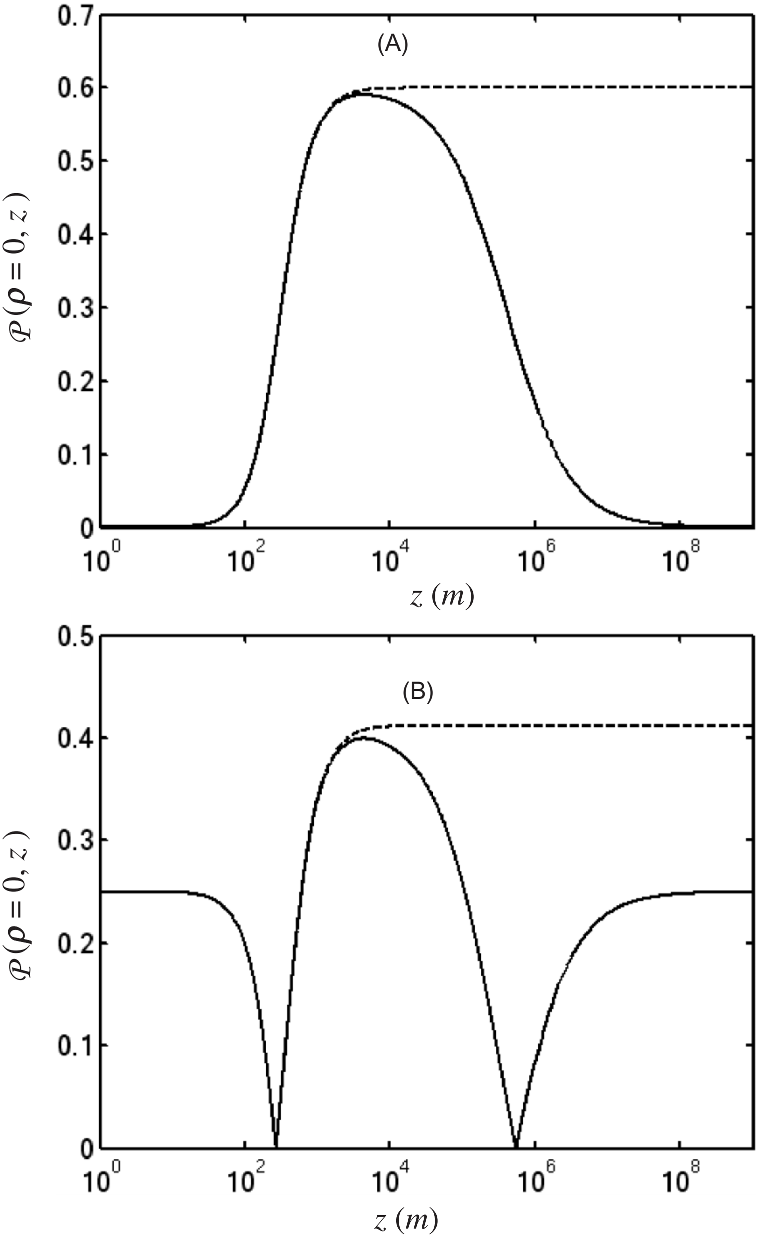

Figure 2. The degree of polarization on the axis of a GSM electromagnetic beam as a function of z for four different values of the initial degree of polarization: (A) $I_x = I_y$; (B)$I_x = (5/3)I_y$; (C)$I_x = 3I_y$; (D)$I_x = 19I_y$. The solid curves correspond to propagation in the turbulent medium. The dashed curves correspond to propagation in free space $(C_n^2 = 0)$ are shown for comparison. The values of the parameters of the beam and turbulence are: $\sigma = 5\,\text{cm}, \delta_{xx} = 0.5\,\text{mm}, \delta_{yy} = 1\,\text{mm}, \eta_{xy}^{(0)} = \eta_{yx}^{(0)} = 0, l_0 = 5\,\text{mm}$, $C_n^2 = 10^{-14}\ \text{m}^{-2/3}$, and $k = 2\pi/\lambda = 10^7$.

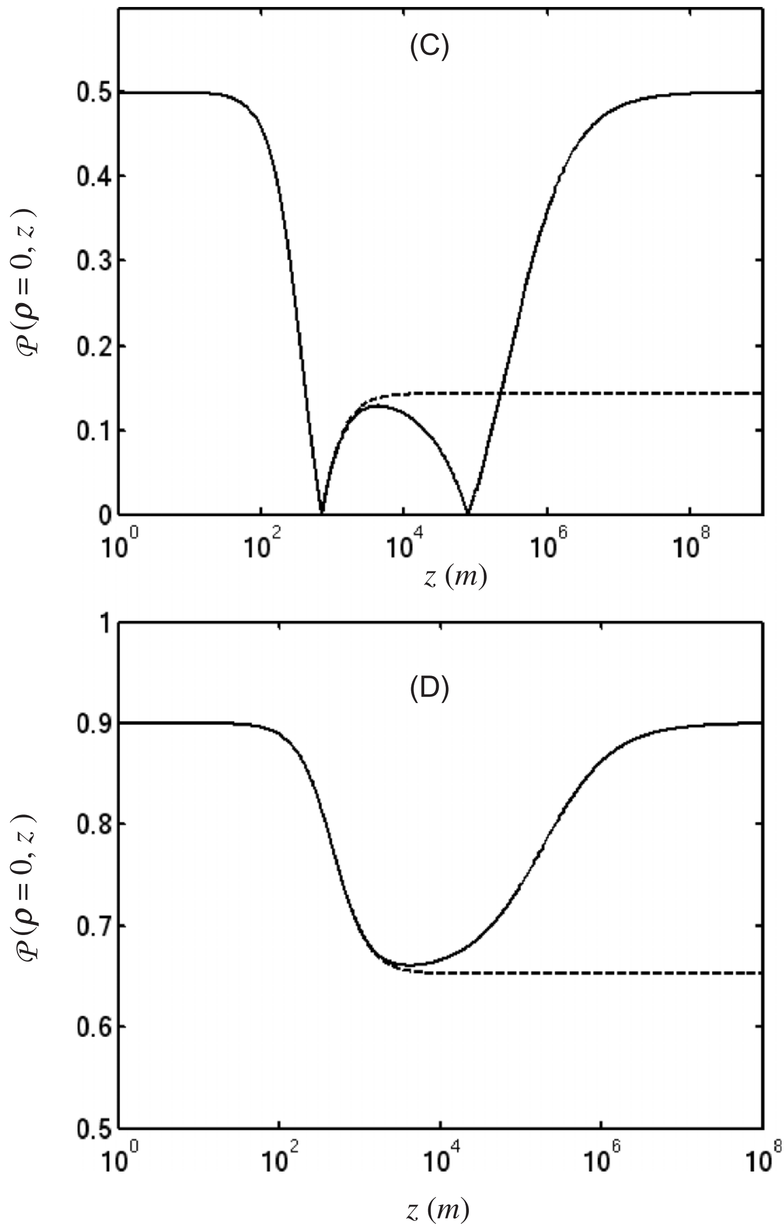

Figure 2. Continued.

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