Degree of phase-space separability of statistical pulses

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Abstract: We introduce the concept of phase-space separability degree of statistical pulses and show how it can be determined using a bi-orthogonal decomposition of the pulse Wigner distribution. We present explicit analytical results for the case of chirped Gaussian Schell-model pulses. We also demonstrate that chirping of the pulsed source serves as a powerful tool to control coherence and phase-space separability of statistical pulses.

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1. Introduction: Wigner function description of statistical pulses

The acute recent interest in femtosecond pulses and pulse sources [1] has triggered resurgence of activity in the field of ultrafast statistical optics [2–11]. The latter should come as no surprise as statistical properties of ultrashort pulses impose ultimate limits on the performance and accuracy of the state-of-art fiber-optical communication systems, for instance; not to mention that spontaneous emission noise degrades the output coherence of optical amplifiers [12]. To date then, there has been extensive research done on modeling statistical properties of realistic sources of ultrashort pulses [2, 3]. The fundamental issues of defining and measuring the spectrum of statistical pulses [4] and formulating coherence theory of periodic statistical pulse trains [5, 6] have been addressed. Due to its relevance for fiber optical communications, the propagation of ultrashort statistical pulses in linear [13, 14] and nonlinear [15] dispersive media has also been explored. Further, several approaches have been recently advanced to synthesize novel partially coherent pulses from uncorrelated–or partially correlated–superpositions of elementary pulses in time [7] and frequency [8]. In addition, several phase-space approaches to partially coherent pulse representation were discussed in the literature [9, 10]. Lately, a general phase-space approach has been put forward to describe partially coherent pulse synthesis from complex Gaussian pulses [11]. A key feature of the complex Gaussian representation is its versatility: It can be used to either generate new partially coherent pulses or represent the ones with known cross-correlation functions in terms of statistically uncorrelated Gaussian pulses.

In this work, we show that a Wigner distribution based phase-space representation for statistical pulses provides a natural context to define a measure of their phase-space separability. Next, we show how the introduced degree of phase-space separability can be determined using a bi-orthogonal decomposition of the Wigner distribution of the pulse. We then discuss the way the new measure changes on chirped pulse propagation in linear dispersive media. Our results are directly relevant for temporal imaging with ultrashort pulses which involves temporal lenses, chirping the pulses, and dispersive delay lines, e. g., linear optical fibers [16]. To make our results more instructive, we specialize to a representative case of chirped Gaussian Schell-model pulses for which closed-form analytical results can be obtained. It follows from our results that chirping ultrashort partially coherent pulses can provide a potent tool for controlling their degrees of coherence and phase-space separability.

To set the stage, we consider an ensemble of random pulses $\{E(t)\}$ and decompose the electric field $E(t)$ into a slowly-varying envelope $U(t)$ and a carrier wave [17] such that

$$E(t) = U(t)e^{-i\omega_c t}, \tag{1}$$

where ω_c is a deterministic carrier frequency of the pulse. The second-order statistical properties of the ensemble $\{U(t)\}$ are specified by the cross-correlation function

$$\Gamma(t_1, t_2) = \langle U^*(t_1)U(t_2) \rangle, \tag{2}$$

where the angle brackets denote ensemble averaging. The Wigner distribution (WD) of the ensemble is then defined as

$$\mathscr{W}(t, \omega) = \int_{-\infty}^{\infty} d\tau \, \Gamma(t - \tau/2, t + \tau/2) e^{-i\omega\tau}, \tag{3}$$

where we introduced the variables $t = (t_1 + t_2)/2$ and $\tau = t_1 - t_2$. The intensity $I(t)$ and spectrum $S(\omega)$ of the pulse are determined by the appropriate marginals of the Wigner distribution viz.,

$$I(t) = \int_{-\infty}^{\infty} d\omega \, \mathscr{W}(t, \omega); \qquad S(\omega) = \int_{-\infty}^{\infty} dt \, \mathscr{W}(t, \omega). \tag{4}$$

Many phase space characteristics of the pulses, such as the position of the pulse center (in time), central frequency of the envelope, rms temporal and spectral widths etc., can be determined as the corresponding moments of I or S. Notice also that it follows at once from Eqs. (2) and (3) that fully as well as partially temporarily and/or spectrally coherent pulses can be treated in the same way using the Wigner distribution, the fully coherent case being just the limiting situation when the cross-correlation function factorizes.

In this work, we will examine the WD evolution as pulses propagate in transparent homogeneous linear dispersive media. To this end, recall that the pulse envelope in such media obeys the paraxial wave equation [18]

$$i\partial_z U - \frac{1}{2}\beta_2 \partial_{ss}^2 U = 0, \tag{5}$$

where we introduced the retarded time $s = t - \beta_1 z$, β_1 being the inverse group velocity, and β_2 is the group-velocity dispersion (GVD) coefficient. It can be inferred at once from Eqs. (2) and (5) that Γ obeys the evolution equation in the form

$$i\partial_z \Gamma + \frac{1}{2}\beta_2 (\partial_{s_1 s_1}^2 - \partial_{s_2 s_2}^2)\Gamma = 0. \tag{6}$$

Thus, the WD propagation is governed by the first-order equation

$$\partial_z \mathscr{W} + \beta_2 \partial_T \mathscr{W} = 0, \tag{7}$$

where the variables

$$T = \frac{s_1 + s_2}{2}, \quad \text{and} \quad \tau = s_1 - s_2, \tag{8}$$

were introduced. Solving Eq. (7) on characteristics, we obtain

$$\mathscr{W}(T, \omega; z) = \mathscr{W}_0(T - \beta_2 \omega, \omega). \tag{9}$$

In other words, as a statistical pulse propagates in a linear dispersive medium such as an optical fiber, say, its Wigner distribution in any transverse plane $z = const$ can be related to its form in the source plane, $\mathscr{W}_0(t, \omega)$, by a rather simple equation. Equation (9) is a temporal analog of the well-known transformation law describing passage of partially coherent beams through first-order optical systems [19, 20] and is a direct consequence of space-time analogy [21–23] between beam and pulse propagation in free space and linear dispersive media, respectively.

2. Quantifying phase-space separability of statistical pulses

The Wigner distribution contains all available information about second-order statistics of the pulses. This information can be revealed by directly measuring the set of phase-space projections of WD known as a Radon-Wigner transform [24]. In general, however, such measurements are rather involved. The situation drastically simplifies for WDs in the separable form such that

$$\mathscr{W}(t,\omega) \propto I(t)S(\omega). \tag{10}$$

The separability of Eq. (10) implies that the second-order statistics of the system can be recovered by separately measuring the intensity profile and spectrum of the pulse. This observation prompts the question: Are there any realistic pulses with a separable WD? A related fundamental issue is: Given an arbitrary statistical ensemble of pulses, how can one quantitatively describe phase-space separability of its WDs? And to follow up on this: How can one control such separability, if at all?

To address the first question, it is sufficient to recall that a common technique to generate statistical pulses involves temporal chopping of statistically stationary light fields [3]. We can consider, for instance, chopping a Gaussian correlated statistically stationary field with a Gaussian temporal modulation function. This procedure yields the so-called Gaussian Schell-model (GSM) source [2] with the correlation function that can be transformed to

$$\Gamma_{\text{GSM}}(t_1,t_2) = I\left(\frac{t_1+t_2}{2}\right) g(t_1-t_2), \tag{11}$$

where both I and g are Gaussians. It follows at once from Eq. (3) and (11) that the WD of a GSM source has a separable form of Eq. (10). Thus, realistic statistical pulses belonging to a wide GSM class have separable WDs. Moreover, the cross-correlation function of any quasi-stationary source can be well approximated as

$$\Gamma_{\text{qs}}(t_1,t_2) \simeq I\left(\frac{t_1+t_2}{2}\right) \gamma(t_1-t_2), \tag{12}$$

where $I(t)$ is a "slowly-varying" intensity profile and $\gamma(t)$ is a "fast" temporal degree of coherence. The WD of such sources are approximately separable as well.

Unfortunately, even if the WD of a source is separable, the WD of generated pulses need not be so as is seen from Eq. (9). In general, the pulse propagation even in a linear dispersive medium couples phase-space variables, providing additional impetus for our quest for a phase-space separability measure. To address this issue, we propose to expand the pulse WD in any transverse plane into a bi-orthogonal series as

$$\mathscr{W}(T,\omega;z) = \sum_n \lambda_n(z)\chi_n(T,z)\phi_n(\omega,z), \tag{13}$$

where the eigenvalues $\{\lambda_n\}$ and eigenfunctions, $\{\chi_n\}$ and $\{\phi_n\}$, are real due to reality of the WD. It is known [25] that the two sets of eigenfunctions obey the Fredholm integral equations which, in our case, take the form

$$\lambda_n(z)\phi_n(\omega,z) = \int_{-\infty}^{\infty} dT\, \mathscr{W}(T,\omega;z)\phi_n(T,z), \tag{14}$$

and

$$\lambda_n(z)\chi_n(T,z) = \int_{-\infty}^{\infty} d\omega\, \mathscr{W}(T,\omega;z)\chi_n(\omega,z). \tag{15}$$

The suitably normalized eigenfunctions form an orthonormal set in the sense that

$$\int_{-\infty}^{\infty} dx\, \chi_n(x,z)\phi_m(x,z) = \delta_{nm}, \qquad x = T, \omega. \tag{16}$$

We note that as the WD may take on negative values, the eigenvalues in the expansion (13) can be negative as well. The latter circumstance distinguishes Eq. (13) from the conventional coherent-mode decomposition of the cross-correlation function [26]. Next, one can arrange the squares of the eigenvalues in decreasing order, $\lambda_0^2 \geq \lambda_1^2 \geq \lambda_2^2 \geq \ldots$. Introducing the reduced eigenvalues $v_n(z)$'s such that $v_n^2 = \lambda_n^2/\lambda_0^2 \leq 1$, we can define the degree of phase-space separability of the pulse by the expression

$$\rho(z) = \frac{1}{\sum_{n=0}^{\infty} v_n^2(z)}. \tag{17}$$

It follows from the definition that the degree of separability is bound by unity, $0 \leq \rho(z) \leq 1$, attaining its maximum if there is only the lowest-order eigenvalue present in the expansion (13). This corresponds to the ideal case of complete separability of the WD. Thus the proposed measure conforms to our intuitive perception of the degree of separability.

3. Degree of separability of chirped Gaussian Schell-model (CGSM) pulses

We will now illustrate the introduced concept using a particular example of chirped Gaussian Schell-model pulses. Not only does the latter serve as a rather representative case, but it enables us to obtain closed-form analytical results. Moreover, we can show explicitly in the case of CGSM pulses the way to control the WD's degree of separability. CGSM pulses can be generated, for example, by transmitting GSM pulses through a time lens which imposes a quadratic phase chirp on the pulse [21].

The cross-correlation function of CGSM pulses can be written in the form

$$\Gamma_{\text{CGSM}}(t_1, t_2) = \Gamma_{00} \exp\left[-\frac{(1-iC)t_1^2}{2t_p^2} - \frac{(1+iC)t_2^2}{2t_p^2} - \frac{(t_1-t_2)^2}{2t_c^2}\right], \tag{18}$$

where t_p is a characteristic pulse width, t_c is a pulse coherence time, C is a dimensionless chirp parameter, and Γ_{00} is a normalization constant. Using the definition of WD (3), we can express the WD of a CGSM pulse as

$$\mathscr{W}_{\text{CGSM}}(t, \omega) = \mathscr{W}_{00} \exp\left[-\frac{t^2}{t_p^2}\left(1 + \frac{C^2 t_{\text{eff}}^2}{2t_p^2}\right) - \frac{\omega^2 t_{\text{eff}}^2}{2} + \frac{C t_{\text{eff}}^2}{t_p^2}\omega t\right], \tag{19}$$

where

$$\frac{1}{t_{\text{eff}}^2} = \frac{1}{t_p^2} + \frac{1}{2t_c^2}. \tag{20}$$

It can be readily inferred from Eq. (19) that because of phase chirping, the WD of a CGSM source is not separable.

The WD of a pulse generated by a CGSM source can be determined from Eqs. (7) and (19); the result can be represented as

$$\mathscr{W}_{\text{CGSM}}(T, \omega; z) = \mathscr{W}_{00} \exp\left[-\frac{T^2}{t_p^2}\left(1 + \frac{C^2 t_{\text{eff}}^2}{2t_p^2}\right) - \frac{\omega^2 \sigma^2(z) t_{\text{eff}}^2}{2} + \frac{C(z) t_{\text{eff}}^2}{t_p^2}\omega T\right]. \tag{21}$$
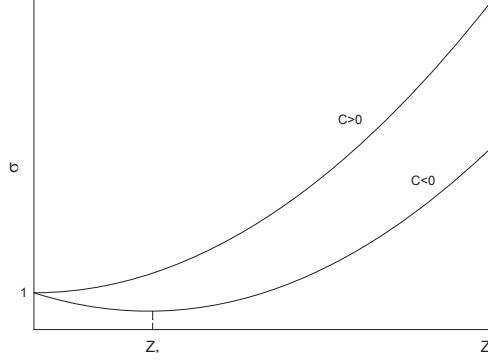
Fig. 1. Sketching the behavior of the propagation factor of a CGSM pulse as a function of dimensionless propagation distance $Z = \beta_2 z/t_p$.

Here we introduced the propagation factor $\sigma(z)$ given by

$$\sigma^2(z) = \left(1 + \frac{C\beta_2 z}{t_p^2}\right)^2 + \frac{2\beta_2^2 z^2}{t_p^2 t_{\text{eff}}^2}, \tag{22}$$

and the effective chirp parameter $C(z)$ as

$$C(z) = C + \frac{\beta_2 z}{t_p^2}\left(C^2 + \frac{2t_p^2}{t_{\text{eff}}^2}\right). \tag{23}$$

Eqs. (22) and (23) are generalizations to the case of partially coherent pulses of the corresponding expressions for fully coherent chirped Gaussian pulses [18]. The latter can be recovered from the former by letting $t_{\text{eff}} = \sqrt{2}t_p$ which corresponds to $t_c \to \infty$. The evolution scenario of σ depends on the sign of $C\beta_2$. If $C\beta_2 \geq 0$, σ increases monotonously with the propagation distance. On the other hand, if $C\beta_2 < 0$, the propagation factor attains a minimum,

$$\sigma_{\text{min}} = \sqrt{\frac{2t_p^2/t_{\text{eff}}^2}{C^2 + 2t_p^2/t_{\text{eff}}^2}}, \tag{24}$$

at the distance

$$z_* = -\frac{Ct_{\text{eff}}^2/\beta_2}{C^2 + 2t_p^2/t_{\text{eff}}^2}. \tag{25}$$

This behavior is qualitatively sketched in Fig. 1. We also note that at z_* the effective chirp is equal to zero–the accrued chirp on propagation in the medium unchirps the initial chirp of the opposite sign imposed by the time lens.

Next, we can conclude by comparing Eqs. (19) and (21) that the WD maintains its Gaussian shape, with the spectral part and the coupling term–the last term in the exponential function in Eq. (21)–scaling on propagation. Alternatively, we can work out the cross-correlation function by taking a Fourier transform of the WD with respect to ω. We arrive, after straightforward algebra, at the expression

$$\Gamma_{\text{CGSM}}(s_1, s_2; z) = \frac{\Gamma_{00}}{\sigma(z)} \exp\left[-\frac{(s_1^2 + s_2^2)}{2t_p^2\sigma^2(z)} - \frac{(s_1 - s_2)^2}{2t_c^2\sigma^2(z)} + \frac{iC(z)}{2t_p^2\sigma^2(z)}(s_1^2 - s_2^2)\right]. \tag{26}$$

It is clear from Eq. (26) that the propagation factor σ governs the dynamics of the pulse width and coherence time.

To determine the degree of separability, we use the following Mehler's summation formula for Hermite polynomials [27]

$$\exp\left(-\frac{x^2+y^2-2xy\zeta}{1-\zeta^2}\right) = \sqrt{1+\zeta^2}e^{-x^2-y^2}\sum_{n=0}^{\infty}\frac{\zeta^n}{2^n n!}H_n(x)H_n(y), \tag{27}$$

where $|\zeta| \leq 1$. Next, we introduce the scaling factors $a(z)$ and $b(z)$ such that in the scaled variables $\tilde{T} = t/a(z)$ and $\tilde{\omega} = \omega/b(z)$, the WD of a CGSM takes the form

$$\mathscr{W}_{\mathrm{CGSM}}(\tilde{T},\tilde{\omega};z) = \mathscr{W}_{00}\exp\left(-\frac{\tilde{T}^2+\tilde{\omega}^2-2\tilde{T}\tilde{\omega}\zeta(z)}{1-\zeta^2(z)}\right). \tag{28}$$

On comparing Eqs. (28) and (27), we conclude that the WD in the original variables can be expanded into a bi-orthogonal series (13) with the eigenvalues

$$\lambda_n(z) = \mathscr{W}_{00}\sqrt{[1+\zeta^2(z)]a(z)b(z)}\,\zeta^{2n}(z), \tag{29}$$

and the eigenfunctions

$$\chi_n(T,z) = \frac{1}{\sqrt{2^n n!\sqrt{\pi}a(z)}}e^{-T^2/a^2(z)}H_n\left[\frac{T}{a(z)}\right], \tag{30}$$

and

$$\phi_n(\omega,z) = \frac{1}{\sqrt{2^n n!\sqrt{\pi}b(z)}}e^{-\omega^2/b^2(z)}H_n\left[\frac{\omega}{b(z)}\right]. \tag{31}$$

In Eqs. (30) and (31), the scaling factors are given by the expressions

$$a(z) = \sqrt{\frac{2t_p^4/t_{\mathrm{eff}}^2}{(C^2+2t_p^2/t_{\mathrm{eff}}^2)[1-\zeta^2(z)]}}, \tag{32}$$

and

$$b(z) = \sqrt{\frac{2}{\sigma^2(z)[1-\zeta^2(z)]t_{\mathrm{eff}}^2}}, \tag{33}$$

where

$$\zeta^2(z) = \frac{C^2(z)}{\sigma^2(z)(C^2+2t_p^2/t_{\mathrm{eff}}^2)}, \tag{34}$$

and positive roots are assumed in Eqs. (32) and (33). We note in passing that the bi-orthogonal decomposition (13) should not be confused with the more familiar coherent-mode expansion of a GSM source [28, 29]. In our case, the eigenfunction sets $\{\chi_n\}$ and $\{\phi_n\}$ belong to different domains which is formally reflected in quite different scaling of a and b with the propagation distance.

Using Eqs. (34) and (29) in Eq. (17), we can show that the degree of separability turns out to be given by a remarkably simple expression

$$\rho(z) = \sigma_{min}^2/\sigma^2(z). \tag{35}$$

The analysis of Eq. (35) reveals that the behavior of ρ as a function of the propagation distance qualitatively depends on the sign of the initial chirp. If $C \geq 0$, the degree of phase separability
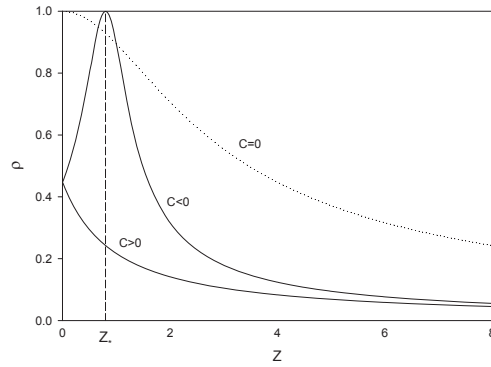
Fig. 2. Degree of phase-space separability of a fully coherent CGSM pulse as a function of dimensionless propagation distance $Z = \beta_2 z / t_p$ for three values of the initial chirp: $C = 0$ and $C = \pm 1$.
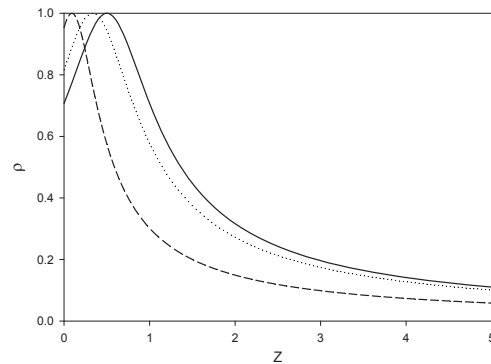

Fig. 3. Degree of phase-space separability of a partially coherent CGSM pulse as a function of dimensionless propagation distance $Z = \beta_2 z / t_p$; solid, $t_c = \infty$, dotted, $t_c = t_p$ and dashed $t_c = \sqrt{2} t_p / 3$. The initial chirp is $C = -1$.

monotonously decreases with the propagation distance. If, on the other hand, $C < 0$, ρ goes through a maximum attained at $z = z_*$. These scenarios are exhibited in Fig. 2 where we present the behavior of ρ as a function of dimensionless propagation distance in the coherent case using three values of the chirp, $C = 0$ and $C = \pm 1$ for illustration. The influence of pulse coherence time on the evolution of ρ is illustrated in Fig. 3 for several values of t_c and $C = -1$. Interestingly, the WD in the CGSM case becomes separable precisely at the distance where the pulses are the most compressed and least coherent. Chirping GSM pulses with a time lens then provides a powerful tool to control pulse coherence and phase-space separability at the same time.

We stress in conclusion that although quantitative details of the presented decomposition are specific of the CGSM pulses, qualitatively the results are quite general and entirely independent of a particular source model. Our findings can be summarized by saying that (i) the introduced degree of phase-space separability of statistical pulses is intimately related with their propagation characteristics in linear dispersive media and (ii) initial chirping, e. g., with a time lens, makes it possible to effectively control the degree of phase-space separability of the pulses.