

Self-induced transparency quadratic solitons

Soodeh Haghgoo and Sergey A. Ponomarenko*

Department of Electrical and Computer Engineering, Dalhousie University,
Halifax, NS, B3J 2X4 Canada

*serpo@dal.ca

Abstract: We discover and theoretically explore self-induced transparency quadratic solitons (SIT-QS) supported by the media with quadratic optical nonlinearities, doped with resonant impurities. The fundamental frequency of input pulses is assumed to be close to the impurity resonance. We envision an ensemble of inhomogeneously broadened semiconductor quantum dots (QD) in the strong confinement regime grown on a substrate with a quadratic nonlinearity to be a promising candidate for the laboratory realization of SIT-QS. We also examine the influence of inhomogeneous broadening as well as wave number and group-velocity mismatches on the salient properties of the introduced solitons.

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1. Introduction

The solitons have been at the heart of nonlinear physics for more than three decades. The reason being they emerge as ultimate survivors of virtually any wave evolution scenario in conservative nonlinear systems of unprecedented physical diversity, from hydrodynamic [1, 2] and plasma waves [2] to pulses in optical fibers [3] and matter waves in Bose-Einstein condensates [3,4]. In the optical context in particular, the interest in the quadratic solitons (QS), which are coupled self-trapped entities in the media with quadratic nonlinearities, has been recently reinvigorated thanks to the advances in fabrication of phase matching structures, see Ref. [5] for a review.

Although the history behind QS can be traced back to 1967 when Ostrovskii discovered self-induced phase changes in nonlinearly coupled fundamental wave (FW) and second harmonic (SH) in a noncentrosymmetric crystal [6, 7], the QS were not experimentally observed for a while. As soon as high-damage optical materials with long enough interaction lengths became available, however, spatial QS were observed in the bulk $\chi^{(2)}$ media [8] and in planar $\chi^{(2)}$ waveguides [9].

On the other hand, most common quadratic materials have very low group-velocity dispersion (GVD) and fairly large group-velocity mismatch (GVM). Thus, extremely long samples and/or short pulses are required to explore the quadratic soliton regime [5]. These obstacles hampered temporal QS generation until the application of an ingenious technique for tailoring GVD and GVM culminated in the experimental observation of their key signatures [10]. Yet, in the pioneering experiment [10] the nonlinear crystal was too short and pulses too long to unequivocally observe a dispersion-free propagation regime over several dispersion lengths. Thus, the existence of alternative routes to temporal QS formation—not relying on balancing $\chi^{(2)}$ nonlinearities with the GVD—is a highly relevant question, which, to our knowledge, has not yet been adequately addressed. We note, however, that in the three-wave interaction situation in $\chi^{(2)}$ -media, neutrally stable QS were discovered [11] which do not rely on the GVD for their existence, but require three waves with different carrier frequencies to mix.

In this paper, we present a new type of QS, self-induced transparency quadratic solitons (SIT-QS), formed in resonant $\chi^{(2)}$ media. The SIT-QS generation does not at all require any bulk

medium GVD, thereby circumventing the major obstacle for temporal $\chi^{(2)}$ soliton generation. The proposed SIT-QS have a hybrid nature: the mutual self-trapping of FW and SH is achieved by the joint action of both resonant nonlinearity, induced by exciting one-exciton transitions in QDs, and the quadratic nonlinearity of the bulk medium. Specifically, we assume that an input FW pulse is nearly resonant with one-exciton transitions in QDs, grown and randomly spread over a substrate exhibiting a quadratic nonlinear optical response. As the FW pulse enters the medium, it coherently excites the QD ensemble and—provided the pulse has a right area—it can open a “transparency window” in the medium leading to the fundamental soliton formation via the SIT phenomenon [12]. At the same time, the SH is generated via the $\chi^{(2)}$ nonlinearity, provided a characteristic resonant absorption length is of the same order as a typical length associated with the bulk $\chi^{(2)}$ nonlinearity. The SH pulse is “dragged” by the FW into the transparency window forming a mutually self-trapped SIT-QS pair. We stress that our results reveal a hitherto unknown extension of quantum optical SIT solitons to the realm of quadratic solitons in $\chi^{(2)}$ media.

Next, we emphasize that the SIT-QS formation demands neither large GVD nor extremely high input pulse intensity—and hence ultrashort duration—which sets them apart from the conventional temporal QS in quadratic media. Further, the discovered SIT-QS are fundamentally different from the previously reported small-amplitude quasisolitons supported by resonant media with quadratic nonlinearities [13]. First, while the latter exist for large wave number mismatch, the SIT-QS require rather accurate phase matching which can be realized in quasi-phase-matched structures. More importantly, though, the QS of Ref. [13] are supported by media with collective plasmonic resonances which, being far from saturation, can be modeled by the classical anharmonic oscillator model. In contrast in the SIT-QS case, the FW interaction with an ensemble of individual QD resonances involves a swift time evolution of the QD level populations which can only be adequately described quantum mechanically.

2. Quantitative analysis and physical model

We now present our quantitative model. We begin by examining short pulse propagation in a $\chi^{(2)}$ medium, randomly doped with QDs. The dopants can be modeled as two-level systems. The fundamental pulse carrier frequency ω is assumed to be very close to the resonance frequency ω_0 of a one-exciton transition in a QD; bi-exciton transitions in the QD ensemble are forbidden by restricting ourselves to the case of circularly polarized FW and SH pulses [14]. Thus, only is the FW resonantly coupled with the exciton transition, while the SH is only coupled with the FW via the $\chi^{(2)}$ nonlinearity. Provided the cubic nonlinear effects are negligible [15], the slowly varying field envelopes \mathcal{E}_1 and \mathcal{E}_2 of the fundamental and second harmonic pulses, respectively, obey the coupled wave equations

$$\left(\frac{\partial}{\partial z} + \frac{1}{v_{g1}} \frac{\partial}{\partial t} \right) \mathcal{E}_1 = \frac{iNd_{eg}\omega^2}{2k_1\epsilon_0c^2} \langle \sigma \rangle - \frac{i\omega^2\chi_{eff}^{(2)*}}{2k_1c^2} \mathcal{E}_1^* \mathcal{E}_2 e^{i\Delta kz}, \quad (1)$$

$$\left(\frac{\partial}{\partial z} + \frac{1}{v_{g2}} \frac{\partial}{\partial t} \right) \mathcal{E}_2 = -\frac{i\omega^2\chi_{eff}^{(2)}}{k_2c^2} \mathcal{E}_1^2 e^{-i\Delta kz}. \quad (2)$$

Here v_{g1} (v_{g2}) is a group velocity of the fundamental (second harmonic) pulse, $\chi_{eff}^{(2)}$ is an effective second-order susceptibility of the bulk medium, $\Delta k = 2k_1 - k_2$ is a wave number mismatch factor, N is a density of dopants, d_{eg} is a dipole matrix element of the ground-to-excited state QD transition – the states are appropriately labeled by the indices g and e – and σ is a quantum dipole envelope function describing the temporal evolution of the QD dipole moment. Further, the quantum dipole moment σ and inversion w envelope functions obey the Bloch equations

which can be written in the circular polarization basis as [12, 16]

$$\partial_\tau \sigma = -(\gamma_\perp + i\Delta)\sigma - i\Omega_1 w, \quad (3)$$

$$\partial_\tau w = -\gamma_\parallel(w - w_{eq}) - \frac{i}{2}(\Omega_1^* \sigma - \Omega_1 \sigma^*). \quad (4)$$

Here we introduced the Rabi frequencies, $\Omega_{1,2} = 2d_{eg}\mathcal{E}_{1,2}/\hbar$ of the FW and SH as well as the transverse (dipole) and longitudinal (energy) relaxation rates as γ_\perp and γ_\parallel , respectively.

Hereafter, it will prove convenient to transform to the reference frame moving with the fundamental pulse by changing variables viz., $\tau = t - z/v_{g1}$ and $\zeta = z$. In the dimensionless variables, $Z = \zeta/L_A$, $T = \tau/\tau_p$ and $\bar{\Omega}_{1,2} = \Omega_{1,2}\tau_p$, the coupled wave equations take the form

$$\frac{\partial \bar{\Omega}_1}{\partial Z} = i \langle \sigma \rangle - \frac{iL_A}{4L_{NL}} \bar{\Omega}_1^* \bar{\Omega}_2 e^{i(\delta Z + \phi_0)}, \quad (5)$$

and

$$\left(\frac{\partial}{\partial Z} + \frac{s_\pm L_A}{L_W} \frac{\partial}{\partial T} \right) \bar{\Omega}_2 = -\frac{ik_1 L_A}{2k_2 L_{NL}} \bar{\Omega}_1^2 e^{-i(\delta Z + \phi_0)}. \quad (6)$$

Here τ_p is a characteristic duration of the input FW pulse, $\delta = \Delta k L_A$ is the dimensionless wave number mismatch, ϕ_0 is a constant relative phase of the effective susceptibility and the dipole matrix element, and $s_\pm = +1(-1)$ for $v > 0 (< 0)$ specifies the sign of the group-velocity mismatch (GVM), $v = v_{g2}^{-1} - v_{g1}^{-1}$. Further, we introduced three key physical length scales to this problem: the linear absorption length L_A , the walk-off length L_W associated with the group velocity mismatch (GVM) of the fundamental and second-harmonic pulses, and the characteristic nonlinear length L_{NL} in the $\chi^{(2)}$ medium defined as

$$L_A = \frac{1}{\alpha}; \quad L_W = \frac{\tau_p}{|v|}; \quad L_{NL} = \frac{k_1 c^2 |d_{eg}| \tau_p}{\hbar \omega^2 |\chi_{eff}^{(2)}|}, \quad (7)$$

where $\alpha = N|d_{eg}|^2 \omega^2 / \sqrt{2\pi} \hbar b k_1 \epsilon_0 c^2$ is a linear absorption coefficient, and b is a spectral width of inhomogeneous broadening.

Next, the average over a distribution of dimensionless frequency detunings from one-exciton resonances in QDs, $\bar{\Delta} = (\omega - \omega_0)\tau_p$, is defined as

$$\langle \sigma \rangle \equiv \int d\bar{\Delta} g(\bar{\Delta}) \sigma(\bar{\Delta}). \quad (8)$$

In this work, we assume the inhomogeneous broadening distribution to be a generic Gaussian function in the form

$$g(\bar{\Delta}) = \frac{1}{\sqrt{2\pi} b \tau_p} \exp\left(-\frac{\bar{\Delta}^2}{2b^2 \tau_p^2}\right). \quad (9)$$

In the coherent transient regime we explore, the pulses must be short enough: $\tau_p \ll \min(T_\perp, T_\parallel)$, where $T_\perp = \gamma_\perp^{-1}$ and $T_\parallel = \gamma_\parallel^{-1}$ are the transverse and longitudinal relaxation times. Thus, the Bloch Eqs. (3) and (4) reduce to

$$\partial_T \sigma = -i\bar{\Delta} \sigma - i\bar{\Omega}_1 w, \quad (10)$$

$$\partial_T w = -\frac{i}{2}(\bar{\Omega}_1^* \sigma - \bar{\Omega}_1 \sigma^*). \quad (11)$$

In our numerical simulations, we consider In(Ga)As QDs grown on a GaAs substrate as a prototype for our system. GaAs is known to have a sizable second-order nonlinearity, $\chi_{eff}^{(2)} \simeq$

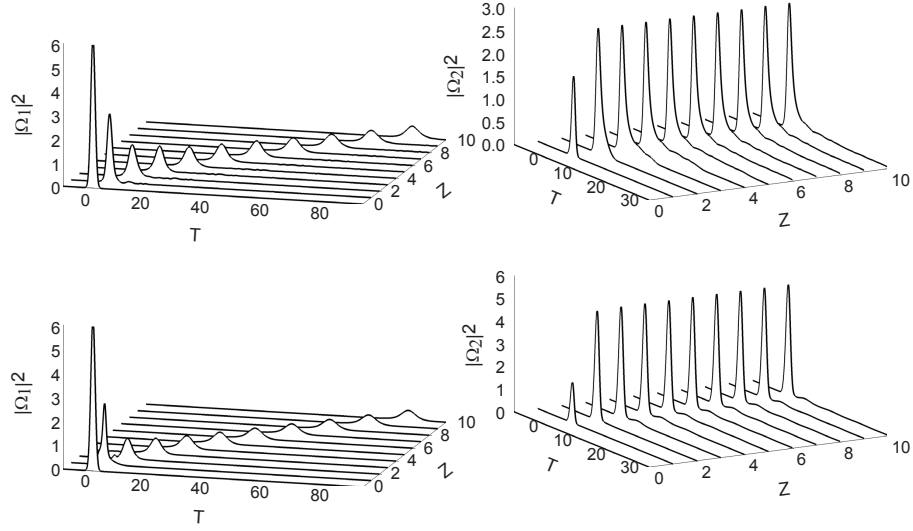


Fig. 1. Top row: intensity profiles of the FW and SH soliton pair. The parameters are $b = 10^7$ MHz, $\delta = 0$, and $L_W = \infty$. Bottom row: same as the top row except there is no inhomogeneous broadening.

2×10^{-10} m/V [17]. Moreover, high efficiency SHG was experimentally demonstrated in the medium thanks to efficient quasi-phase matching in orientation-patterned GaAs samples [18, 19]. We take $N \simeq 10^{21} \text{ m}^{-3}$, the dipole moment matrix element to be $d_{eg} \simeq 5 \times 10^{-29}$ Cm, all magnitudes consistent with recent simulations [20, 21] and experimental work [21, 22].

Further, we work with the input 2π -pulses in the picosecond range, say, $\tau_p \simeq 1$ ps which is also consistent with the previous experimental and numerical work on SIT on bound [21] and free [23] exciton resonances. As the characteristic relaxation times for typical QDs are $T_{\perp} \simeq 2T_{\parallel} \sim 100$ ps at room temperature—and are several times longer if the sample is cooled down to cryogenic temperatures [24]—the pulses are well within the coherent transient regime. The inhomogeneous broadening width depends on the ensemble preparation; a typical estimate runs as $b \simeq 10^7$ MHz [20]. Using the above numerical values, we estimate $L_A \simeq 1.5$ mm and $L_{NL} \simeq 0.8$ mm, implying that $L_A \sim L_{NL}$ which is favorable for the SIT-QS pair formation. Taking into account that v is typically in the range 0.1-1 ps/mm for most $\chi^{(2)}$ crystals [25], we estimate L_W to fall in the range from 1 mm to 1 cm.

3. Numerical simulations

A qualitative analysis—borne out by subsequent numerical simulations—reveals that the SIT-QS formation and stability crucially depend on the magnitudes of the four factors: inhomogeneous broadening width, wave number mismatch, temporal walk-off length, and the area of the incident fundamental pulse. In our numerical simulations using Eqs. (5) and (6) as well as Eqs. (8)–(11), we consider a 2π -Gaussian input FW pulse entering the medium; we assume there is no power in the SH at $Z = 0$. To illustrate the SIT-QS formation in the presence of inhomogeneous broadening, we exhibit in Fig. 1 the evolution of the FW and SH pulse intensities as functions of the propagation distance Z . For simplicity, we assume perfect phase matching and no temporal walk-off. It is seen in the figure that a stable SIT-QS pair emerges over two characteristic propagation distances for an inhomogeneously broadened sample with $b = 10^7$ MHz (top row of Fig. 1). We stress that we have numerically propagated the solitons over several dozens of

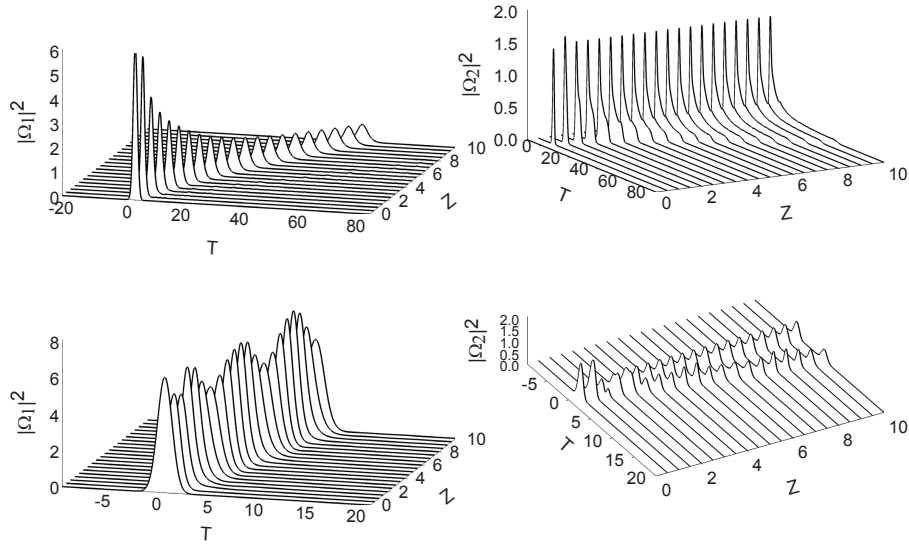


Fig. 2. Top row: intensity profiles of the FW and SH soliton pair. The parameters are $b = 10^7$ MHz, $\delta = 3$, and $L_W = \infty$. Bottom row: same as above except $b = 0$.

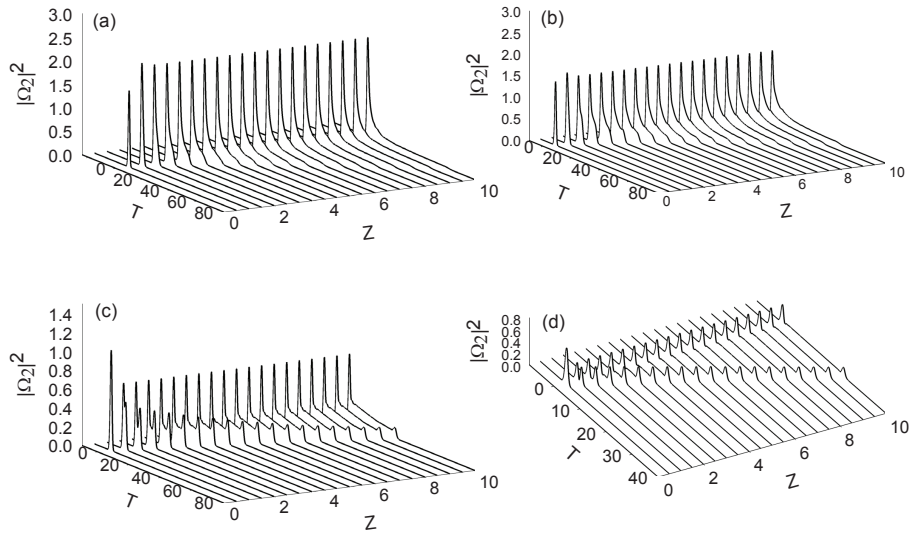


Fig. 3. Intensity profile of the SH wave for (a) $\delta = 2$, (b) $\delta = 3$, (c) $\delta = 5$ and (d) $\delta = 7$. The other parameters are $b = 10^7$ MHz and $L_W = \infty$.

characteristic absorption lengths and found no trace of instability. For comparison, the SIT-QS pair formation is also displayed for an ideal homogeneously broadened sample in the bottom row of Fig. 1. We observe that as the resonant light-QD interaction efficiency decreases due to the inhomogeneous broadening—which is always present for realistic QD samples—the second-harmonic soliton pulse develops a tail at the trailing edge.

Next, we examine the role of inhomogeneous broadening in the SIT-QS formation with a finite phase mismatch. In Fig. 2 we compare the SIT-QS pair formation in the presence of the

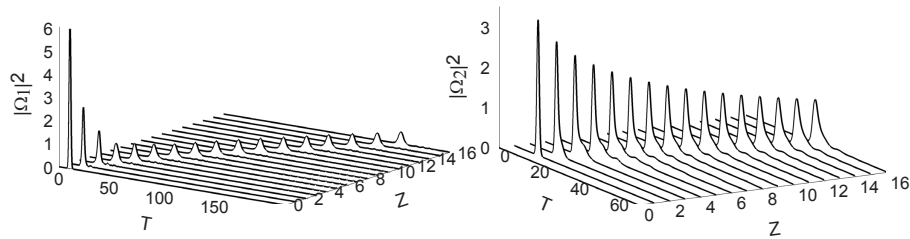


Fig. 4. Intensity profiles of the FW and SH soliton components. The parameters are $b = 10^7$ MHz, $\delta = 1$, and $L_W = 1$ mm.

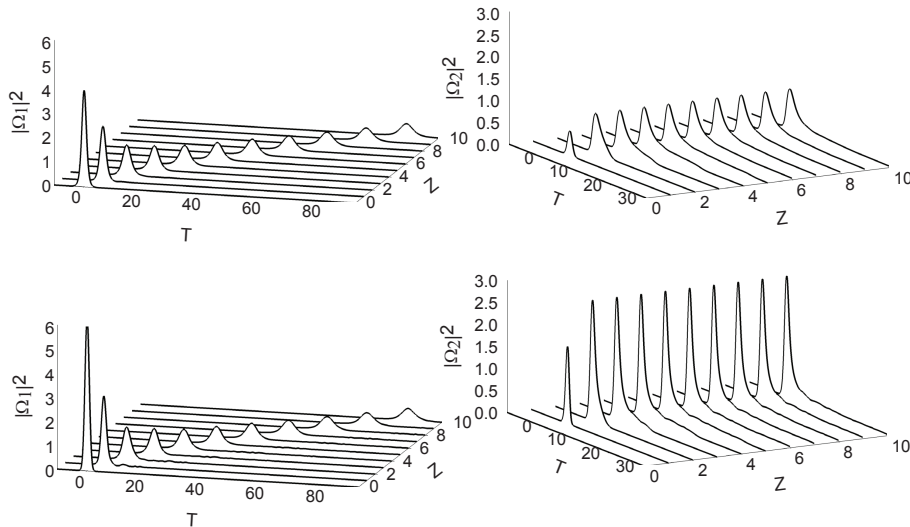


Fig. 5. Intensity profiles of the FW and SH soliton pair for secant hyperbolic (top row) and Gaussian (bottom row) input 2π fundamental pulses. The other parameters are $b = 10^7$ MHz, $\delta = 0$ and $L_W = \infty$.

wave number mismatch, $\delta = 3$ in homogeneously and inhomogeneously broadened samples. It is seen in the figure that in a perfectly clean sample even a moderate phase mismatch causes breathing of the FW and splitting of the SH pulses, respectively. In contrast, the inhomogeneous broadening inhibits the instability and promotes stable SIT-QS pair generation albeit with the SH component developing a tail at the trailing edge.

The tail, however, grows rather slowly (adiabatically) on propagation, implying that the SH component can be interpreted as a quasi-stable soliton even in the moderate phase mismatch situation. As the phase mismatch increases, though, the inhomogeneous broadening can no longer prevent the SH component break-up into two small-amplitude pulses. This situation is clearly illustrated in Fig. 3 where the SH component intensity profile is displayed for four values of the dimensionless wave number mismatch. Thus, we conclude that while the inhomogeneous broadening plays a stabilizing role, the lack of the perfect phase matching can result in the internal dynamics of the SIT-QS leading to their eventual disintegration.

Further, we study the temporal walk-off influence on the SIT-QS dynamics. In Fig. 4 we plot the SIT-QS pair formation for the case of an inhomogeneously broadened sample with a modest wave number mismatch, $\delta = 1$ and with the walk-off length of 1 mm. As is seen in the figure,

a stable SIT-QS pair is generated, although the peak amplitude of the SH component decreases on the soliton propagation as the overlap between the FW and SH is reduced due to walk-off. Thus the SIT-QS formation is possible even in the presence of a rather substantial temporal walk-off.

Next, we performed a number of numerical simulations with the input FWs of various temporal profiles, including Gaussian and secant hyperbolic. The latter corresponds to a fundamental SIT soliton in the FW component. Our simulations indicate that salient features of the SIT-QS formation are largely insensitive to the initial temporal profile of the fundamental pulse. To illustrate this point, we present in Fig. 5 a comparison of the SIT-QS evolution for the 2π -secant hyperbolic (top row) and Gaussian (bottom row) input pulses for the perfectly phase-matched situation with zero GVM, for simplicity. One can see in the figure that although the efficiency of SHG with secant hyperbolic pulses is reduced, the key qualitative features of the SIT-QS pair persist. In physical terms, the different SHG efficiency is explained by the fact that powers of the input 2π Gaussian and secant hyperbolic pulses are quite different, of course.

On the other hand, the SIT-QS evolution scenario strongly depends on the input FW area, $\mathcal{A} \equiv \int_{-\infty}^{\infty} \Omega_1 d\tau$. In fact, we discovered that no SIT-QS formation is possible whenever the magnitude of \mathcal{A} is below π . For any input FW pulse with the area, $\pi \leq \mathcal{A} \leq 2\pi$, though, the area magnitude goes through several oscillatory cycles, asymptotically attaining the value of 2π ; the same holds true for pulses with the area greater than 2π . The oscillatory transient dynamics of the area distinguishes the SIT-QS case from the pure SIT case in which the area monotonously approaches 2π [12].

4. Conclusion

In conclusion, we have discovered and numerically explored a new type of solitons, self-induced transparency quadratic solitons supported by noncentrosymmetric nonlinear media, doped with resonant impurities. We have shown that the SIT-QS evolution and stability does not depend on the shape of the input FW, but are strongly influenced by the interplay of inhomogeneous broadening and phase mismatch in the system. Whereas the former is a stabilizing factor, the latter causes the instability that can result in the QS disintegration. To experimentally realize the envisioned SIT-QS, we recommend that relatively long—a few centimeter long, say—quasi-phase-matched samples be used. We have also demonstrated that the GVM does not substantially affect the SIT-QS formation as long as the samples are sufficiently short. To mitigate—to the point of eliminating—the GVM effects in long samples, one can use tilted input pulses, following the technique of Ref. [10]. Finally, we note that although we considered only quadratic bulk nonlinearities in the manuscript [15], the third-order nonlinearities can become important for femtosecond input pulses and/or sufficiently large phase mismatch. The resulting competition between $\chi^{(2)}$ - and $\chi^{(3)}$ -effects, similar to those discussed in Ref. [26], can lead to new interesting soliton regimes in the system. We plan to address this issue in the new context of SIT-QSs in a forthcoming publication.