Complex area correlation theorem for statistical pulses in coherent linear absorbers

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We derive a complex area correlation theorem describing global second-order statistical properties of pulses propagating in coherent linear absorbers. We also illustrate temporal evolution of a generic partially coherent pulse in a coherent linear absorber by discussing the behavior of its temporal intensity profile and degree of coherence.

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As statistical properties of ultrashort pulses impose ultimate limits on the performance and accuracy of the state-of-art fiber-optical communication systems [1], there has been acute interest in exploring the evolution of statistical pulses in a variety of dispersive media that can serve as conduits for optical communications. In this context, the dynamics of partially coherent pulses in generic, weakly dispersive linear media was explored [2,3]. More specifically, coherence and polarization properties of such pulses were discussed on their propagation in optical fibers [4,5], and statistical properties of certain classes of partially coherent pulses propagating in linear and nonlinear dispersive media were examined and characterized in novel ways [6,7].

More recently, the interest has arisen to statistical pulse propagation in resonant media. In particular, it was demonstrated that any asymmetric pulse with a sharp leading edge evolves toward a universal self-similar asymptotic shape on propagation in coherent linear amplifiers near optical resonance [8]. In coherent linear absorbers, on the other hand, self-similarity can be generated only for specific–although rather wide–classes of input coherent [9] and partially coherent [10] pulses. The research to date, however, leaves unexplored the propagation properties of generic partially coherent pulses in resonant linear absorbers and amplifiers.

In this Letter, we examine global and local statistical properties of generic ultrashort pulses propagating in linear absorbers in a resonant regime. We demonstrate the existence of a novel theorem, the area correlation theorem, which governs the **universal behavior** of complex area correlations of any statistical pulse on propagation in a resonant linear absorber. We explain the physical significance of the theorem and relate it to the (generalized) area theorem known to determine the dynamics of coherent pulse area in such media. We also discuss the temporal intensity profile and degree of coherence evolution for Gaussian Schell-model pulses in coherent linear absorbers.

To begin, we examine small-area pulse propagation in a coherent absorber under near resonance condition: the pulse carrier frequency ω_c is tuned closely to a resonant transition frequency ω_0 of the medium atoms. In the slowly-varying envelope approximation (SVEA), the pulse field $\mathcal{E}(z,t)$ and atomic dipole moment $\sigma(z,t)$ can be shown to obey the classical Maxwell–Lorentz equations (MLE) [8,11]

$$\partial_\zeta \Omega = i\kappa\langle\sigma\rangle_\Delta, \tag{1}$$

and

$$\partial_\tau \sigma = -(\gamma_\perp + i\Delta)\sigma + i\Omega, \tag{2}$$

which are written in the transformed variables: $\zeta = z$ and $\tau = t - z/c$. Here $\Delta = \omega_c - \omega_0$ is a detuning of the pulse carrier frequency from resonance and the angle brackets with the subscript "Δ" denote averaging over the frequency detuning distribution $g(\Delta)$, defined as

$$\langle\sigma\rangle_\Delta = \int_{-\infty}^{\infty} \mathrm{d}\Delta\sigma(\tau,\zeta;\Delta)g(\Delta). \tag{3}$$

In Eqs. (1) and (2) we introduced the field envelope in frequency units, $\Omega = -e\mathcal{E}/2m\omega x_0$, where x_0 is an amplitude of the electron displacement from equilibrium, the inverse dipole relaxation rate $\gamma_\perp = 1/T_\perp$, where $T_\perp$ is a characteristic dipole moment relaxation time, and a coupling constant, $\kappa = Ne^2/4\epsilon_0 mc$. The coupled MLE can be solved using a Fourier transform technique, yielding the field envelope at any propagation distance in the form

$$\mathcal{E}(\tau,\zeta) = \int_{-\infty}^{\infty} \mathrm{d}\omega\tilde{\mathcal{E}}_0(\omega)\exp[-i\omega\tau - \kappa\mathcal{R}(\omega)\zeta]. \tag{4}$$

Here the spectral response function of the medium $\mathcal{R}$ and the spectral amplitude of the incident pulse are defined as

$$\mathcal{R}(\omega) = \left\langle \frac{1}{\gamma_\perp + i(\Delta - \omega)} \right\rangle_\Delta. \tag{5}$$

and

$$\tilde{\mathcal{E}}_0(\omega) = \int_{-\infty}^{\infty} \frac{\mathrm{d}t}{2\pi}\mathcal{E}(t,0)e^{i\omega t}, \tag{6}$$

respectively.

Let us now consider an ensemble of statistical realizations of pulses $\{\mathcal{E}(\tau,\zeta)\}$. The second-order statistical

properties of the ensemble are, in general, characterized by the two-time, two-distance correlation function, defined as

$$\Gamma(\tau_1,\zeta_1;\tau_2,\zeta_2)=\langle\mathcal{E}^*(\tau_1,\zeta_1)\mathcal{E}(\tau_2,\zeta_2)\rangle. \tag{7}$$

Here the angle brackets without subscripts denote pulse ensemble averaging. It follows at once from Eqs. (4) and (7) that

$$\Gamma(\tau_1,\zeta_1;\tau_2,\zeta_2)=\int_{-\infty}^{\infty}\mathrm{d}\omega_1\int_{-\infty}^{\infty}\mathrm{d}\omega_2 W_0(\omega_1,\omega_2)\times e^{i(\omega_1\tau_1-\omega_2\tau_2)}\exp\{-\kappa[\mathcal{R}^*(\omega_1)\zeta_1+\mathcal{R}(\omega_2)\zeta_2]\}, \tag{8}$$

where $W_0(\omega_1,\omega_2)$ is the cross-spectral density at the source given by

$$W_0(\omega_1,\omega_2)=\langle\tilde{\mathcal{E}}_0^*(\omega_1)\tilde{\mathcal{E}}_0(\omega_2)\rangle. \tag{9}$$

Next, we introduce a complex area of a statistical pulse by the expression

$$\mathcal{A}(\zeta)\equiv\int_{-\infty}^{\infty}\mathrm{d}\tau\mathcal{E}(\tau,\zeta). \tag{10}$$

Equation (10) is a generalization of the conventional real area under unchirped pulse—see [11]—to the case when a statistical pulse is chirped at the source; of course, the complex area does lose a direct geometrical interpretation as the area under the pulse temporal profile. Moreover, $\mathcal{A}$ pertains to a member of the statistical ensemble. Hence, it is a random function of the propagation distance. One can then introduce the area correlation function viz.,

$$\mathcal{C}_{\mathcal{A}}(\zeta_1,\zeta_2)\equiv\langle\mathcal{A}^*(\zeta_1)\mathcal{A}(\zeta_2)\rangle=\int_{-\infty}^{\infty}\mathrm{d}\tau_1\int_{-\infty}^{\infty}\mathrm{d}\tau_2\Gamma(\tau_1,\zeta_1;\tau_2,\zeta_2). \tag{11}$$

Further, on taking the double time integration on both sides of Eq. (8), employing the area correlation definition (11) and the integral representation of the delta function,

$$\delta(\omega)=\int_{-\infty}^{\infty}\frac{\mathrm{d}t}{2\pi}e^{i\omega t}, \tag{12}$$

we arrive at

$$\mathcal{C}_{\mathcal{A}}(\zeta_1,\zeta_2)=\mathcal{C}_{\mathcal{A}0}\exp\{-\kappa[\mathcal{R}^*(0)\zeta_1+\mathcal{R}(0)\zeta_2]\}, \tag{13}$$

where $\mathcal{C}_{\mathcal{A}0}=\mathcal{C}_{\mathcal{A}}(0,0)$. Equation (13) can be transformed, with the aid of Eq. (5), to the final form of the correlation area theorem as

$$\mathcal{C}_{\mathcal{A}}(\zeta_1,\zeta_2)=\mathcal{C}_{\mathcal{A}0}e^{-\alpha(\zeta_1+\zeta_2)}e^{i\beta(\zeta_2-\zeta_1)}, \tag{14}$$

where we introduced a small-signal absorption coefficient α and the phase accumulation factor β by the expressions

$$\alpha=\left\langle\frac{2\kappa\gamma_\perp}{\gamma_\perp^2+\Delta^2}\right\rangle_\Delta, \tag{15}$$

and

$$\beta=\left\langle\frac{2\kappa\Delta}{\gamma_\perp^2+\Delta^2}\right\rangle_\Delta. \tag{16}$$

Equation (14) is the key result of this Letter. It tells us that due to absorption, the area correlations of a statistical pulse exponentially decay on propagation into the coherent linear absorber, regardless of a specific temporal profile of the pulse at the source. Notice that in the same transverse plane, $\zeta_1=\zeta_2$, the area correlation function does not pick up any additional phase on propagation–it simply exponentially decays with the propagation distance ζ. We also notice that in the fully coherent limit, the area correlation function factorizes and the derived area correlation theorem reduces to

$$\mathcal{A}(\zeta)=\mathcal{A}_0e^{-\alpha\zeta}e^{i\beta\zeta}, \tag{17}$$

which is a generalized area theorem for coherent pulses. The generalization entails the extension of the area concept of [11] to chirped pulses, thereby allowing for a complex area. The presence of the phase factor on the r. h. s. of Eq. (17) is a consequence of pulse chirping by the medium; in the form (14)–or (17) for fully coherent pulses–the classical area theorem applies to chirped small-area pulses as well.

To study local properties of statistical pulses on their propagation in resonant linear absorbers, we must specialize to a particular pulse model. In this work, we consider a Gaussian Schell-model (GSM) statistical pulse as a rather representative case. The cross-spectral density of a GSM pulse can be expressed as [12]

$$W_0(\omega_1,\omega_2)\propto\exp\left[-\frac{(\omega_1-\omega_2)^2t_p^2}{2}\right]\exp\left[-\frac{(\omega_1+\omega_2)^2t_{\mathrm{eff}}^2}{8}\right], \tag{18}$$

where

$$\frac{1}{t_{\mathrm{eff}}^2}=\frac{1}{t_c^2}+\frac{1}{4t_p^2}, \tag{19}$$

t_p and t_c being the characteristic pulse width and coherence time, respectively. To proceed further we need to specify $g(\Delta)$. Any choice will depend on the physical nature of inhomogeneous broadening. To examine a generic case, we choose $g(\Delta)$ to be a (normalized) Lorenzian function,

$$g(\Delta)=\frac{1}{\pi}\frac{1/T_\Delta}{\Delta^2+1/T_\Delta^2}, \tag{20}$$

where T_Δ is a characteristic damping time associated with inhomogeneous broadening. This choice ensures that the spectral response function can be determined in the explicit form as [11]

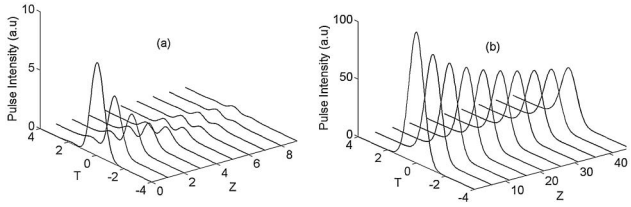

Fig. 1. GSM pulse intensity profile. The pulse parameters are (a) $t_c = T_{\text{eff}} = 5t_p$ and (b) $t_c = T_{\text{eff}} = t_p/5$.

$$\mathcal{R}_L(\omega) = \frac{1}{1/T_{\text{eff}} + i(\Delta - \omega)}. \tag{21}$$

Here $T_{\text{eff}}^{-1} = T_{\perp}^{-1} + T_{\Delta}^{-1}$ is the effective damping time.

We numerically explore the behavior of the GSM pulse intensity profile, $I(\tau, \zeta) = \Gamma(\tau, \zeta; \tau, \zeta)$, on propagation in the absorber. To this end, we transform to dimensionless variables, $T = t/T_{\text{eff}}$, $Z = \alpha\zeta$, and measure the intensity in arbitrary units (a.u). The pulse coherence state will affect its intensity evolution in the resonant absorber whenever $t_c \sim T_{\text{eff}}$. It follows from Eqs. (18) and (19) that the inverse of $\min(t_c, t_p)$ plays the role of the effective input pulse bandwidth. In this connection, we can distinguish two characteristic cases: very "short", $\min(t_c, t_p) \ll T_{\text{eff}}$, and very "long", $\min(t_c, t_p) \gg T_{\text{eff}}$, pulses.

In Fig. 1 we present the evolution of the GSM pulse intensity profile as a function of Z for (a) very short, $t_c = T_{\text{eff}} = 5t_p$, and (b) very long, $t_c = T_{\text{eff}} = t_p/5$, pulses. It is seen in the figure that the short pulse, which has a broad spectrum as compared with the spectral amplitude $\mathcal{R}_L(\omega)$ of the absorbing medium, is strongly absorbed and reshaped, transferring its remaining energy toward its tails. The long pulse, on the other hand, keeps its shape almost intact, but its peak intensity decays on propagation.

Next, we exhibit in Figs. 2 and 3 the evolution of the magnitude of the temporal degree of coherence, defined as

$$\gamma(\tau_1, \zeta; \tau_2, \zeta) \equiv \frac{\Gamma(\tau_1, \zeta; \tau_2, \zeta)}{\sqrt{I(\tau_1, \zeta) I(\tau_2, \zeta)}}, \tag{22}$$

for relatively long and rather short pulses, respectively. It is seen in the figures that while the degree of coherence of a long pulse is rather weakly affected by the absorbing medium, the coherence state of the short one becomes progressively more inhomogeneous across the pulse profile such that $|\gamma|$ oscillates rapidly between zero and unity. This profound coherence state evolution testifies to a strong coherent coupling of short—and hence spectrally broad—statistical pulses with the resonant absorber atoms.

In summary, we studied global and local correlation properties of statistical pulses propagating in linear absorbing media in the near resonance regime. We have derived a correlation area theorem describing the universal behavior of global correlation properties of the pulse propagating in the medium. We also examined the evolution of intensity profiles and temporal degrees of coherence of Gaussian Schell-model pulses in coherent linear absorbers. We have shown that the intensity profiles and degrees of coherence of short GSM pulses are strongly affected by the medium due to their efficient coupling to the absorbing atoms.

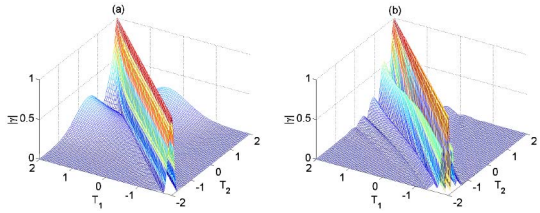

Fig. 2. (Color online) Magnitude of the temporal degree of coherence of a relatively long GSM pulse at (a) $Z = 1$ and (b) $Z = 50$. The pulse parameters are $t_c = T_{\text{eff}} = t_p/5$.

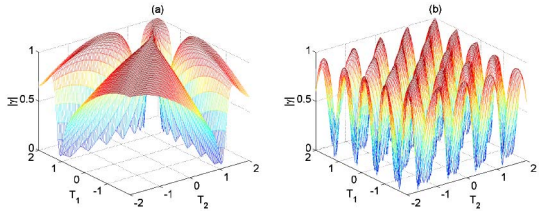

Fig. 3. (Color online) Magnitude of the temporal degree of coherence of a rather short GSM pulse at (a) $Z = 1$ and (b) $Z = 50$. The pulse parameters are $t_c = T_{\text{eff}} = 5t_p$.

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