Optics Communications 286 (2013) 344–346

Contents lists available at SciVerse ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Optical shocks in resonant media: The role of inhomogeneous broadening

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ARTICLE INFO

Article history:
Received 8 July 2012
Received in revised form
24 August 2012
Accepted 3 September 2012
Available online 15 September 2012

Keywords:
Optical shocks
Resonant nonlinear media
Inhomogeneous broadening
Two-level atoms

ABSTRACT

We discover and numerically describe optical shock wave formation in inhomogeneously broadened resonant nonlinear media. Our results extend our previous work to the case of inhomogeneously broadened two-level media. We also describe in detail the atomic variable behavior as the optical shocks form in the medium.

1. Introduction

Shock waves have been discovered in a variety of physical systems, including fluids and gases [1,2]. In general, shock waves are generated by sudden and violent changes in pressure, density, and/or temperature. In most fluid systems, the energy of shocks dissipates due to the viscous damping in the medium through which it travels. However, systems such as cold plasmas [3,4], superfluids and Bose–Einstein condensates [5–8], where the viscosity and damping effects are negligible, support dispersive shock waves.

Shock waves in optics have also been examined by invoking the analogy between superfluid and nonlinear optical wave behaviors [9,10]. The analogy has been explored in Ref. [11] to explain the observed behavior of dispersive optical shocks – which are the optical equivalent of condensate shock waves – in the spatial domain. Dispersive optical shocks have also been observed in the temporal domain using ultrashort pulses in optical fibers [12–15].

At the same time, much less attention has been devoted to optical shocks in resonant nonlinear media. Nevertheless, we have shown elsewhere [16] that in homogeneously broadened resonant nonlinear absorbers, optical shocks are formed as intermediate self-similar asymptotics of any incident pulse with a long tail in the trailing edge. In particular, such shocks can be generated in solids, doped with resonant impurities, and bulk semiconductors, doped with quantum dots. However, in the systems we discussed in Ref. [16], the inhomogeneous broadening plays an important role in shaping the input pulse and hence it cannot be really ignored.

In this paper, we explore the possibility of shock-like pulse formation in resonant nonlinear media in the presence of inhomogeneous broadening. We show that similar to our previous work [16], the interplay between the optical nonlinearity and the transverse relaxation processes – which are responsible for the temporal width of the shocks – cause self-steepening of the input pulse and shock formation. On the other hand, the longitudinal relaxation processes lead to decay and eventual disappearance of the shocks. We stress that the discovered optical shocks form in the pulse envelope, with their characteristic width being determined by the dipole relaxation time. The latter is much longer than an optical cycle, thereby justifying the use of the slowly varying envelope approximation.

2. Mathematical preliminaries and physical model

We model the resonant medium as a two-level system with the resonance frequency ω_0. We assume that the transverse (dipole) relaxation time $T_\perp$ is much shorter than the longitudinal (energy) one, $T_\parallel$, yet much longer than an optical cycle

$$\omega_0^{-1} \ll T_\perp \ll T_\parallel. \tag{1}$$

The inequality (1) implies that the atomic dipole moments evolve much faster than the atomic population dynamics unfolds. By this assumption, the shocks are formed by atomic dipole evolution over a much shorter distance than that over which energy dissipation takes its toll. We consider solids, doped with resonant atoms [17], or bulk semiconductors, doped with quantum dots [18] as particular realizations of the system. The characteristic

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http://dx.doi.org/10.1016/j.optcom.2012.09.004

transverse and longitudinal relaxation times for solids doped with resonant atom impurities, fall into the ranges $10^{-6} \le T_{\parallel} \le 10^{-3}$ and $10^{-13} \le T_{\perp} \le 10^{-11}$ s [19], respectively, implying $10^{5} \le T_{\parallel}/T_{\perp} \le 10^{10}$. Also, relaxation times for bulk semiconductors doped with quantum dots range as follows: $10^{-12} \le T_{\parallel} \le 10^{-4}$ and $10^{-13} \le T_{\perp} \le 10^{-12}$ s [19] such that $1 \le T_{\parallel}/T_{\perp} \le 10^{9}$. Therefore, our results are applicable to many physical systems, including all solids and some semiconductor systems.

We consider a light pulse with a carrier frequency ω near the optical resonance frequency ω_0 of a two-level atom medium. We also assume that the pulse spectrum is mainly affected by inhomogeneous broadening. Under these conditions, the slowly varying field envelope of the pulse in terms of the transformed coordinate and time, $\zeta = z$ and $\tau = t - z/c$, obeys the reduced wave equation

$$\frac{\partial \Omega}{\partial \zeta} = \frac{\omega N |d_{eg}|^2}{c\epsilon_0 \hbar} \langle \sigma \rangle. \tag{2}$$

Here $\Omega = 2d_{eg}\mathcal{E}/\hbar$ is the Rabi frequency associated with the pulse amplitude $\mathcal{E}$, N is a density of impurity atoms, d_{eg} is a dipole matrix element between the ground and excited states of any atom; the two relevant atomic states are appropriately labeled with the indices g and e, and σ is a dipole envelope function of atomic dipole moment. The average over a distribution of frequency detunings from atomic resonances, $\Delta = \omega - \omega_0$, is defined as

$$\langle \sigma \rangle \equiv \int d\Delta g(\Delta)\sigma(\Delta). \tag{3}$$

In this work, we assume the inhomogeneous broadening distribution to be a generic Gaussian function in the form

$$g(\Delta) = \frac{1}{\sqrt{2\pi}\delta} \exp\left(-\frac{\Delta^2}{2\delta^2}\right), \tag{4}$$

where δ is a spectral width of inhomogeneous broadening. The quantum dipole moment $\sigma = u - iv$ and inversion w envelope functions obey the Bloch equations which can be written as [17]

$$\partial_\tau \sigma = -(\gamma_\perp + i\Delta)\sigma - i\Omega w, \tag{5}$$

$$\partial_\tau w = -\gamma_{\parallel}(w - w_{eq}) - \frac{i}{2}(\Omega^* \sigma - \Omega \sigma^*). \tag{6}$$

Here $\gamma_\perp$ and $\gamma_{\parallel}$ are defined as the corresponding inverse transverse relaxation time and energy relaxation time, $T_\perp$ and $T_{\parallel}$, respectively. Since a constant background intensity at the trailing edge of the input wave is required to produce a shock, we then consider as an initial condition, a Q-switched laser input of the form

$$\Omega(0,t) = \frac{\Omega_0}{1 + e^{-t/\tau_p}}, \tag{7}$$

where Ω_0 is the amplitude – measured in frequency units – of the cw laser field and τ_p is a characteristic time constant of the switching process. Hereafter, it will prove convenient to introduce dimensionless variables as $T = \gamma_\perp \tau$, $Z = \alpha\zeta$; $\alpha = kN|d_{eg}|^2/\sqrt{2\pi}\epsilon_0\delta\hbar$, being a linear absorption coefficient, $\overline{\Omega} = \Omega/\gamma_\perp$, $\overline{\Delta} = \Delta/\gamma_\perp$, and recast Eqs. (2)–(7).

3. Numerical simulations

We then numerically solve Eq. (2), together with the Bloch Eqs. (5) and (6), subject to the initial condition (7). The simulations reveal the existence of a shock-like solution for the Rabi frequency. In Fig. 1, we show shock formation for sufficiently long energy relaxation times, $T_{\parallel}/T_\perp = 10^7$, and the other parameters such that $\delta = 10^{12}$ s^{-1}, and $T_\perp = 10^{-13}$ s. A fast self-steepening

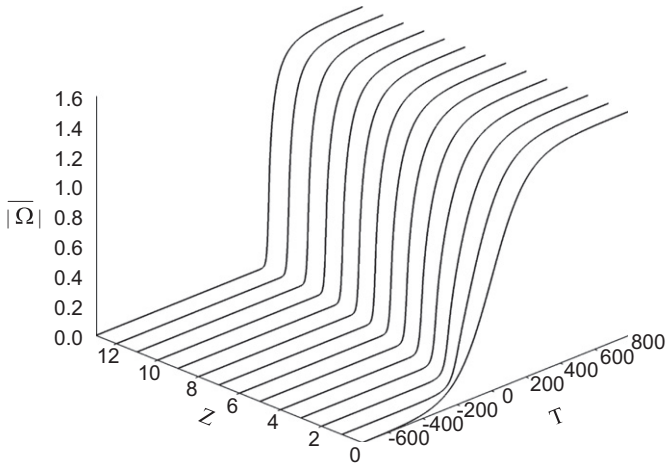

Fig. 1. Dimensionless Rabi frequency $\overline{\Omega}$ of a forming shock as a function of dimensionless time, T, and propagation distance Z. The ratio of transverse to longitudinal relaxation times is $T_\perp/T_{\parallel} = 10^{-7}$. The initial parameters are $\Omega_0 = 1.5\gamma_\perp$, and $\tau_p = 100T_\perp$.

stage results in the steady-state shock formation. It can be inferred from the figure that the shock structure, which is determined by the magnitude of the Rabi frequency jump Ω_0, becomes steep as it propagates in the medium.

Our numerical simulations show that to form a shock, Ω_0 may not be less than a certain critical value Ω_c, which depends on the magnitude of $\gamma_{\parallel}$. The presence of a critical power threshold for shock formation is necessary because the incident wave should have enough power to overcome energy losses due to longitudinal relaxation processes. In our case, $T_{\parallel}/T_\perp = 10^7$, the critical amplitude is found to be $\Omega_c = 7 \times 10^6 \gamma_{\parallel} \simeq 7 \times 10^{12}$ s^{-1}. Translating this to real life units, we can estimate the critical intensity required to form a shock, $I_c = \epsilon_0 nc\hbar^2\Omega_c^2/8d_{eg}^2$. Estimating the dipole moment of a typical impurity atom in a solid to be $d_{eg} \simeq 2 \times 10^{-29}$ C m [17] and the refractive index of a bulk solid material as $n \simeq 2$, we arrive at a rough estimate $I_c \simeq 10$ MW/cm^2, which is an order-of-magnitude below the optical breakdown intensity of a typical solid [20]. One would then have to use high power Q-switched lasers to generate the proposed optical shocks.

To better understand shock formation and to elucidate the role of inhomogeneous broadening, the time evolution of the atomic variables, u, v, and w is displayed in Fig. 2 for several values of Δ. First, we observe that at a fixed Z, w evolution mimics that of the shock amplitude. The reason being that at the leading edge of the shock – where its amplitude is low – no atom inversion takes place and w remains close to its initial value, $w_0 = -1$. On the other hand, a large shock amplitude at the trailing edge saturates the medium implying that $w_\infty = 0$. By the same token, the absorptive component v of the atomic dipole moment peaks only at the trailing edge of the w profile where there is a substantial probability – roughly a half – to find an impurity atom in its excited state. Although the behavior of v is independent of the sign of Δ, its peak amplitude strongly depends on the detuning: the magnitude of the peak is reduced precipitously as the detuning from resonance increases. The dispersive component u does depend on the sign of Δ. And, unlike v, it is more pronounced for the atoms that are farther detuned from resonance. The presence of finite u causes pulse chirping which, in turn, results in smoothing out the transition between the shock edges.

Finally, to explain the behavior of the inversion, we display in Fig. 3, the time evolution of inversion for different propagation distances at a fixed frequency detuning. As we can see in the figure, at the leading edge of the shock, where the light intensity is very small, the atomic population is hardly affected by the

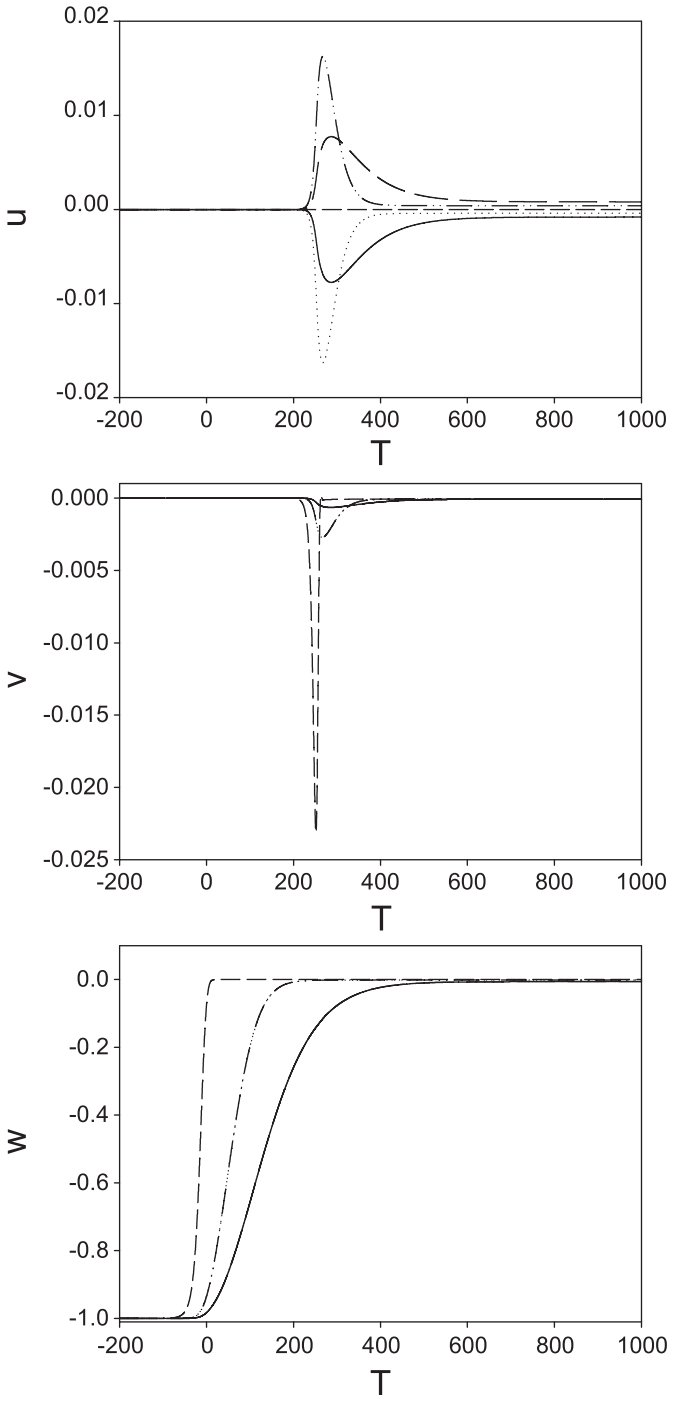

Fig. 2. Atomic dipole moment components (u, v) and one-atom inversion w as functions of dimensionless time, T, displayed at several dimensionless frequency detuning, $\overline{\Delta}$, and at the propagation distance, $Z=13$: solid, $\overline{\Delta}=-12$; dotted, $\overline{\Delta}=-6$; dashed, $\overline{\Delta}=0$; dash-dotted, $\overline{\Delta}=6$; long-dashed, $\overline{\Delta}=12$.

pulse such that the one-atom inversion is approximately given by its equilibrium value, $w_{eq}=-1$. At the trailing edge, however, a large pulse amplitude saturates the medium, implying zero inversion.

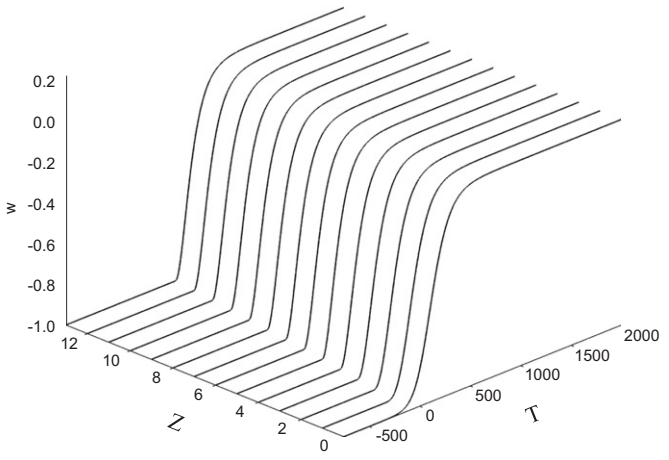

Fig. 3. One-atom inversion w as a function of dimensionless time, T, displayed at several propagation distances, and $\overline{\Delta}=15$. The ratio of transverse to longitudinal relaxation times is $T_\perp/T_\parallel=10^{-7}$. The initial parameters are $\Omega_0=1.5\gamma_\perp$, and $\tau_p=100T_\perp$.

4. Conclusion

In summary, we have numerically shown that the results of our previous work [16] on the novel class of optical shocks in resonant nonlinear media, can be qualitatively extended to the case of inhomogeneously broadened media. The shocks are formed as a result of the interaction between optical nonlinearity and the transverse relaxation processes in the limit of negligible energy relaxation processes.

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