A class of partially coherent beams carrying optical vortices

Sergey A. Ponomarenko

Department of Physics and Astronomy and Rochester Theory Center for Optical Science and Engineering, University of Rochester, Rochester, New York 14627

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A new class of partially coherent beams with a separable phase, which carry optical vortices, is introduced. It is shown that any member of the class can be represented as an incoherent superposition of fully coherent Laguerre–Gauss modes of arbitrary order, with the same azimuthal mode index. The free-space propagation properties of such partially coherent beams are studied analytically, and their M^2 quality factor is investigated numerically. © 2001 Optical Society of America

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1. INTRODUCTION

Since Collett and Wolf[1] demonstrated that a certain class of spatially highly incoherent sources can produce fields as directional as those generated by lasers, there has been a growing interest in studying the properties of partially coherent light sources and the fields that they generate. Among those, the sources producing Gaussian Schell-model (GSM) beams,[2–4] twisted GSM (TGSM) beams,[5–9] and Bessel-correlated Gaussian beams[10,11] as well as some related sources received much attention. GSM beams have been utilized in connection with a speckle reduction problem in diffraction and scattering.[12] It was also demonstrated that the use of partially coherent light produced by a GSM source may enhance the efficiency of certain nonlinear optical processes, most notably the second-harmonic generation.[13] Like fully coherent Bessel–Gauss beams, partially coherent Bessel-correlated Gaussian beams can propagate over large distances with little spreading. This remarkable property suggests the possibility of using such beams in a number of applications.[14]

Recently, light beams possessing wave-front singularities known as optical vortices, have become the focus of many investigations because of their interesting properties[5–9,15–20] as well as because of their potential applications.[21–23] Fully coherent beams with wave-front singularities have been extensively studied.[15–19] However, with the notable exception of TGSM beams and some beams closely related to them,[5–9,20] much less is known about partially coherent beams that may carry optical vortices. To our knowledge, the partially coherent beams carrying optical vortices that have been introduced so far possess a fairly subtle position-dependent phase (twist phase). The twist phase is *inseparable*, i.e., the cross-spectral density of a beam with such a phase at a pair of points in the plane transverse to the direction of propagation cannot be represented as a product of a phase factor and a function depending only on the radial coordinates of the points. On the other hand, the fully coherent beams with optical vortices are the familiar Laguerre–Gauss modes,[24] which have a separable phase with a simple helicoidal structure. The latter circumstance considerably simplifies analysis of such beams. In this connection, it is interesting to ask whether one can construct partially coherent beams with a *separable* phase that carry optical vortices. Beams of this kind could be utilized in the situations where highly isotropic coherence properties are needed. This is so, because, as we will demonstrate, the modulus of the spectral degree of coherence of such beams at any pair of points in a plane transverse to the propagation direction of the beam is *independent* of the relative orientation of the points.

In this paper, beams with such properties are introduced. We will show how the sources that generate these beams can be represented by a combination of the normalized Laguerre–Gauss modes of arbitrary order with the same phase dependence. We then study propagation properties of this new family of beams. Since any member of the family may be constructed from the modes that are shape invariant on paraxial propagation in free space, the shape of the cross-spectral density of any beam that belongs to the new class remains unchanged on propagation as well. We will also find that the cross-spectral density of such beams is invariant with respect to the spatial Fourier transform in the transverse plane, a property that makes these beams similar to GSM and TGSM beams.

This paper is organized as follows. In Section 2, we apply second-order coherence theory in the space-frequency domain to obtain an analytical expression for the cross-spectral density of the new class of sources. In Section 3, the radiant intensity distribution of the fields produced by the new sources is discussed. We then elucidate the conditions under which these sources generate paraxial beams. In Section 4, we find an expression for the cross-spectral density of such a beam at any pair of points in the half-space $z > 0$, into which the beam propagates. Finally, we determine numerically the M^2 quality factor of such beams.

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2. CROSS-SPECTRAL DENSITY OF THE NEW FAMILY OF SOURCES

The goal of this section is to obtain, on the basis of second-order coherence theory in the space-frequency domain (Ref. 25, Sec. 4.7.1), partially coherent fields whose phase structure is similar to that of the fully coherent Laguerre–Gauss beam. For this purpose, we consider such a beam propagating into the half-space $z > 0$.

Let us first recall that the field distribution of such a beam in the source plane $z = 0$ is given by the expression

$$\psi_n^m(\boldsymbol{\rho}) = \left(\frac{\sqrt{2}\rho}{w}\right)^m L_n^m\left(\frac{2\rho^2}{w^2}\right)\exp(-im\phi)\exp\left(-\frac{\rho^2}{w^2}\right), \quad (1)$$

where $\boldsymbol{\rho} = (\rho, \phi)$ is a position vector of a point in the source plane, w is a spot size at the waist of the beam, m is the azimuthal mode index, and n is the order of the Laguerre polynomial $L_n^m(x)$. It is clearly seen from Eq. (1) that the phase dependence of each Laguerre–Gauss mode is specified by a factor $\exp(-im\phi)$; in other words, it has a separable phase.

To find a partially coherent source that generates a field with a separable phase, we recall that the cross-spectral density $W(\boldsymbol{\rho}, \boldsymbol{\rho}', \omega)$ of a partially coherent, planar source can be represented as a Mercer-type series of spatially completely coherent modes $\psi_s(\boldsymbol{\rho}, \omega)$ at given frequency ω by means of the expression (Ref. 25, Sec. 4.7.1)

$$W(\boldsymbol{\rho}, \boldsymbol{\rho}', \omega) = \sum_s \lambda_s \psi_s^*(\boldsymbol{\rho}, \omega)\psi_s(\boldsymbol{\rho}', \omega). \quad (2)$$

Here the subscript s stands for a set of integers labeling the modes, and λ_s is the eigenvalue corresponding to the mode ψ_s.[26] The modes can be chosen to form an orthonormal set. Each mode is a solution of the integral equation

$$\int d^2\rho\, W(\boldsymbol{\rho}, \boldsymbol{\rho}', \omega)\psi_s(\boldsymbol{\rho}) = \lambda_s \psi_s(\boldsymbol{\rho}'), \quad (3)$$

where the eigenvalues λ_s are necessarily real and nonnegative:

$$\lambda_s \geqslant 0. \quad (4)$$

Next, consider the following summation formula for Laguerre polynomials[27]:

$$\sum_{n=0}^{\infty} \frac{n!}{(m+n)!} z^n L_n^m(x) L_n^m(y) = \frac{(xyz)^{-m/2}}{1-z}\exp\left[-\frac{z(x+y)}{1-z}\right] I_m\left(\frac{\sqrt{4xyz}}{1-z}\right). \quad (5)$$

Here $I_m(x)$ is a modified Bessel function of order m, and z is, in general, a complex number such that $|z| < 1$. On making the substitutions $x = 2\rho^2/w^2$, $y = 2\rho'^2/w^2$, and $z = \xi$ and on multiplying both sides of Eq. (5) by $A(\rho/w)^m(\rho'/w)^m \exp[-(\rho^2 + \rho'^2)/w^2]$, we obtain the formula

$$\frac{A\xi^{-m/2}}{1-\xi}\exp[-im(\phi - \phi')] \times \exp\left[-\frac{1+\xi}{1-\xi}\frac{(\rho^2+\rho'^2)}{w^2}\right] I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho\rho'}{w^2}\right) = \sum_{l=-\infty}^{\infty}\sum_{n=0}^{\infty} A\lambda_{nl}\psi_n^{l*}(\boldsymbol{\rho})\psi_n^l(\boldsymbol{\rho}'), \quad (6)$$

where A is a positive constant, $\psi_n^l(\boldsymbol{\rho})$ is given by Eq. (1), and

$$\lambda_{nl} = \frac{n!}{(n+l)!}\xi^l \delta_{ml}, \quad (7)$$

where δ_{ml} is the Kronecker symbol. On comparing Eq. (6) with Eq. (2), we conclude that the former is just a modal expansion of the cross-spectral density of the source that produces a partially coherent field with the separable phase. The modes are the Laguerre–Gauss functions (1), and the eigenvalues are specified by Eq. (7). We rewrite this cross-spectral density in the form

$$W(\boldsymbol{\rho}, \boldsymbol{\rho}') = \frac{A\xi^{-m/2}}{1-\xi}\exp[-im(\phi - \phi')] \times \exp\left[-\frac{1+\xi}{1-\xi}\frac{(\rho^2+\rho'^2)}{w^2}\right] I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho\rho'}{w^2}\right). \quad (8)$$

It should be noted that in view of condition (4), the weight factor ξ is necessarily real and nonnegative; moreover, when we take into account the range of validity of Eq. (5), it follows that $0 < \xi < 1$.

We now briefly examine the spectral intensity and the spectral degree of coherence of a field generated by such a source, and, in doing so, we elucidate the physical significance of the parameter ξ. The expression for the spectral intensity $S(\rho)$, which is represented by the diagonal element of the cross-spectral density, follows at once from Eq. (8):

$$S(\rho) = \frac{A\xi^{-m/2}}{1-\xi}\exp\left(-\frac{2\rho^2}{w^2}\frac{1+\xi}{1-\xi}\right) I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho^2}{w^2}\right). \quad (9)$$

In Fig. 1, a normalized spectral intensity is displayed versus the dimensionless radial coordinate ρ/w. It is seen

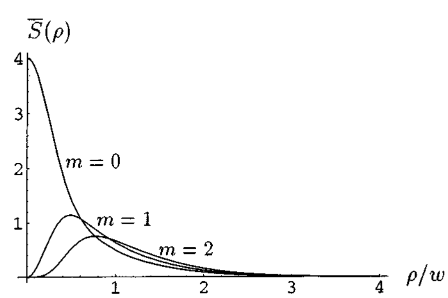

Fig. 1. Normalized spectral intensity $\bar{S}(\rho) = S(\rho)/\int_0^\infty d\rho\, \rho S(\rho)$ versus dimensionless radial variable ρ/w for different values of the azimuthal phase index m ($m = 0, 1, 2$). The coherence parameter ξ is taken to have the value $\xi = 0.5$.

from this figure that any member of the new family of fields, except the axially symmetric one ($m = 0$), has an annular shape with a dark central region. It is of interest to note that the same property is characteristic of another class of partially coherent beams, closely related to the TGSM beams.[20]

The spectral degree of coherence is defined by the expression

$$\mu(\boldsymbol{\rho}, \boldsymbol{\rho}') = \frac{W(\boldsymbol{\rho}, \boldsymbol{\rho}')}{\sqrt{W(\boldsymbol{\rho}, \boldsymbol{\rho})}\sqrt{W(\boldsymbol{\rho}', \boldsymbol{\rho}')}}. \quad (10)$$

In the present case, one finds, with the help of Eq. (8), that

$$\mu(\boldsymbol{\rho}, \boldsymbol{\rho}') = \exp[-im(\phi - \phi')] \times \frac{I_m(2\rho\rho'/{\sigma_c}^2)}{\sqrt{I_m(2\rho^2/{\sigma_c}^2)}\sqrt{I_m(2\rho'^2/{\sigma_c}^2)}}. \quad (11)$$

Here a characteristic distance, the spatial coherence length σ_c, has been introduced, over which the field in the transverse plane remains correlated. It is given by the expression

$$\frac{1}{{\sigma_c}^2} = \frac{2\sqrt{\xi}}{(1 - \xi)w^2}. \quad (12)$$

On solving Eq. (12) for ξ, one arrives at

$$\xi = \frac{{\sigma_c}^4}{w^4}\left[\left(1 + \frac{w^4}{{\sigma_c}^4}\right)^{1/2} - 1\right]^2. \quad (13)$$

It follows from Eq. (13) that ξ specifies the spectral degree of coherence of the field, with $\xi \to 0$ corresponding to the fully coherent case ($\sigma_c \to \infty$) and with $\xi \to 1$ corresponding to the completely incoherent case ($\sigma_c \to 0$). It is also evident from Eq. (13) that $0 < \xi < 1$, which is a necessary condition for the validity of the above mode expansion.

Another important conclusion can be drawn from expression (11): The absolute value of the spectral degree of coherence at a pair of points in the transverse plane is independent of the relative orientation of these points, and it depends solely on the radial distance between them. This conclusion is illustrated in Figs. 2 and 3, where $|\mu|$ has been plotted for the beams with $m = 0$ and $m = 10$, respectively. It can also be seen from these figures that the modulus of the spectral degree of coherence attains its maximum at the points with $\rho = \rho'$ and that the greater the absolute value of the azimuthal phase index m of the beam, the slower the decrease of $|\mu|$ with the radial separation $|\rho - \rho'|$ of the two points.

Finally, on expanding the modified Bessel function of integer order m in a Taylor series for small values of the argument and retaining in Eq. (8) only the leading term,

$$I_m(x) \simeq \frac{1}{m!}\left(\frac{x}{2}\right)^m, \quad (14)$$

one finds that in the fully coherent limit ($\sigma_c \to \infty$), the cross-spectral density reduces to that of the fully coherent, lowest-order Laguerre–Gauss beam:

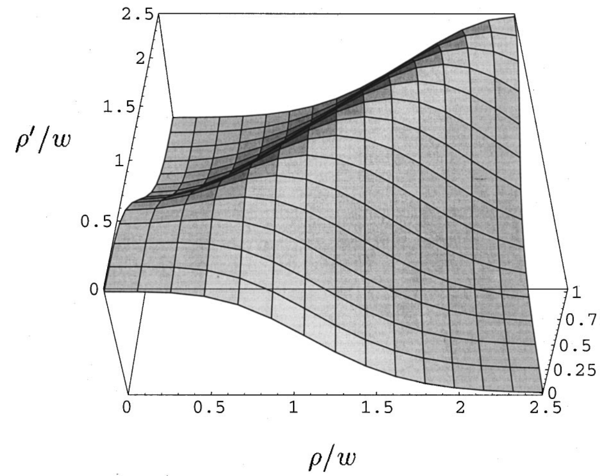

Fig. 2. Modulus of the spectral degree of coherence μ of the axially symmetric field $m = 0$ at a pair of points with polar coordinates (ρ, ϕ) and (ρ', ϕ'), respectively. The coherence parameter ξ is taken to have the value $\xi = 0.5$.

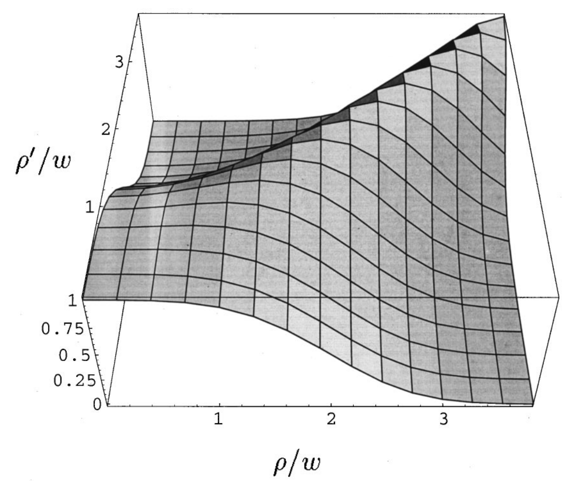

Fig. 3. Same as in Fig. 2 but with the field possessing the azimuthal phase index $m = 10$.

$$W(\boldsymbol{\rho}, \boldsymbol{\rho}') \sim \exp[im(\phi - \phi')]\left(\frac{\rho}{w}\right)^m\left(\frac{\rho'}{w}\right)^m \times \exp\left(-\frac{\rho^2}{w^2}\right)\exp\left(-\frac{\rho'^2}{w^2}\right). \quad (15)$$

3. PROPERTIES OF THE FIELDS GENERATED BY THE NEW CLASS OF SOURCES

A. Radiant Intensity Distribution

In this subsection, we study the radiant intensity distribution of the field generated by the partially coherent source, whose cross-spectral density is given by Eq. (8). The radiant intensity $J(\hat{\mathbf{s}})$ of a statistically stationary, planar, secondary source is defined as the radiative power that crosses a unit area in the far zone, in the direction specified by a unit vector $\hat{\mathbf{s}}$, and characterizes the angular spread of the radiation pattern produced by the source. It can be shown (Ref. 25, Sec. 5.3.1) that it is given by the expression

$$J(\hat{\mathbf{s}}) = \widetilde{W}(-k\mathbf{s}_\perp, k\mathbf{s}_\perp), \tag{16}$$

where $k = \omega/c$ is the wave number associated with the frequency ω; $\mathbf{s}_\perp$ ($|\mathbf{s}_\perp| \leq 1$) is a projection, considered as a two-dimensional vector, of the unit vector $\hat{\mathbf{s}}$ onto the source plane, and $\widetilde{W}(\mathbf{f}, \mathbf{f}')$ is defined by the formula

$$\widetilde{W}(\mathbf{f}', \mathbf{f}) = \int\int \frac{d^2\rho d^2\rho'}{(2\pi)^2} W(\boldsymbol{\rho}, \boldsymbol{\rho}')\exp[-i(\mathbf{f}\boldsymbol{\rho} + \mathbf{f}'\boldsymbol{\rho}')]. \tag{17}$$

Therefore, to find the radiant intensity distribution, one must determine the spatial Fourier transform (17) of the cross-spectral density. Use of the mode expansion considerably facilitates this task. It follows at once from Eq. (8) that

$$\widetilde{W}(\mathbf{f}', \mathbf{f}) = \sum_{n,l} \lambda_{nl} \widetilde{\psi}_n^{l*}(\mathbf{f}')\widetilde{\psi}_n^l(\mathbf{f}), \tag{18}$$

where $\widetilde{\psi}_n^l(\mathbf{f})$ represents the two-dimensional spatial Fourier transform of the mode function (1).

Next, we evaluate the Fourier transform of an individual mode by making use of the integral representation $J_m(x)$ for a Bessel function of order m,[27]

$$J_m(x) = \int_0^{2\pi} \frac{d\phi}{2\pi} \exp(im\phi - ix\cos\phi), \tag{19}$$

and also of the formula[28]

$$\int_0^\infty dx\, x^{\alpha/2} \exp(-px) J_\alpha(b\sqrt{x}) L_n^\alpha(cx) = \left(\frac{b}{2}\right)^\alpha \frac{(p-c)^n}{p^{n+\alpha+1}} \exp\left(-\frac{b^2}{4p}\right) L_n^\alpha\left(\frac{b^2c/4p}{c-p}\right). \tag{20}$$

We carry out the calculation with the parameters $p = 1/2$, $c = 1$, and $b = fw/\sqrt{2}$. On comparing the resulting expression with Eq. (1), we conclude that each mode maintains its functional form upon the spatial Fourier transformation in the transverse plane. Hence the functional form of the cross-spectral density of the field itself is *invariant* under this transformation. The cross-spectral density of the partially coherent field in the Fourier space is then obtained by adding up contributions from the individual modes. The resulting expression is

$$\widetilde{W}(\mathbf{f}, \mathbf{f}') = \frac{A(w^2/2)^2 \xi^{-m/2}}{1-\xi} \exp[im(\theta - \theta')] \times \exp\left[-\frac{1+\xi}{1-\xi} \frac{(f^2 + f'^2)w^2}{4}\right] I_m\left(\frac{\sqrt{\xi} ff' w^2}{1-\xi}\right). \tag{21}$$

Here $\mathbf{f} = (f, \theta)$ and $\mathbf{f}' = (f', \theta')$ are the spatial-frequency vectors in the Fourier space. It now follows from expression (16) for the radiant intensity that

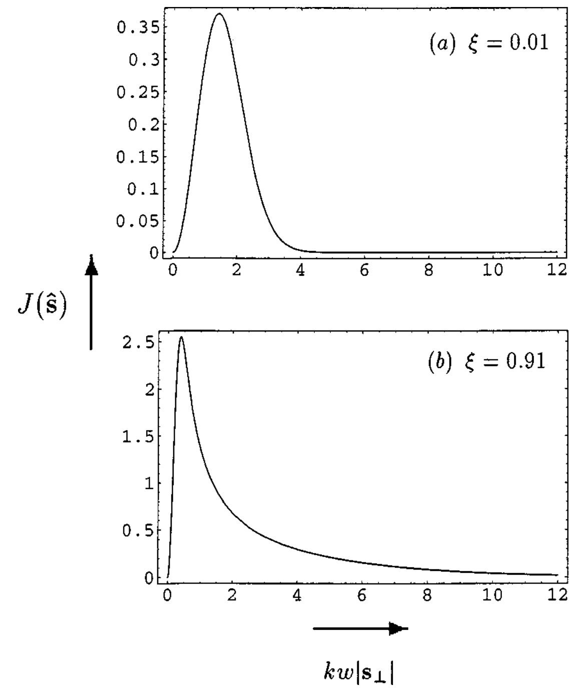

Fig. 4. Radiant intensity of the field with the azimuthal phase index $m = 1$, plotted as a function of the dimensionless spatial frequency $kw|\mathbf{s}_\perp|$ for two cases: (a) very coherent source ($\xi = 0.01$) and (b) nearly incoherent source ($\xi = 0.91$). The radiant intensity is normalized to the total power.

$$J(\hat{\mathbf{s}}) = \frac{B\xi^{-m/2}}{1-\xi} \times \exp\left(-\frac{1+\xi}{1-\xi} \frac{k^2w^2|\mathbf{s}_\perp|^2}{2}\right) I_m\left(\frac{\sqrt{\xi} k^2 w^2 |\mathbf{s}_\perp|^2}{1-\xi}\right), \tag{22}$$

where $B = Aw^4/4$. Figure 4 shows the behavior of the radiant intensity in two limits: a spatially very coherent source ($\xi \approx 0.01$) and a spatially very incoherent one ($\xi \approx 0.91$). It can be seen from the figure that the radiant intensity distribution in the nearly coherent case is fairly symmetric about its maximum, with the distribution width being well approximated by the inverse spot size w; however, the radiant intensity of the nearly incoherent source tends to have a long tail, whose length is roughly equal to the inverse coherence length σ_c. We will show in Subsection 3.B that these qualitative features of the radiant intensity give rise to the appropriate conditions for a source to generate a beamlike field.

B. Generation of a Beamlike Field

So far, we have described the source that produces a field whose cross-spectral density in the source plane is given by Eq. (8), without resorting to the paraxial approximation. However, for such a field to be beamlike, propagating close to the z axis, certain restrictions on the values of the spot size and the spatial coherence length must be imposed. We recall that the radiant intensity $J(\hat{\mathbf{s}})$ of a beamlike field can have appreciable value only in those directions whose unit vectors $\hat{\mathbf{s}}$ form a narrow solid angle around the z axis. Mathematically, this condition can be stated as (Ref. 25, Sec. 5.6.3)

$$\tilde{W}(-\mathbf{f}, \mathbf{f}) \approx 0 \qquad \text{unless } |\mathbf{f}| \ll k. \tag{23}$$

The preceding analysis has revealed that the radiant intensity distribution varies significantly when the state of coherence of the source is varied. It is, therefore, instructive to study separately the cases of a very coherent source and a very incoherent source.

First, we consider an almost coherent source ($\xi \to 0$). In Eq. (21) we expand the modified Bessel function of integer order m in a Taylor series for small values of the argument and keep only the leading term, given by relation (14). Further, on substituting the resulting expression for the radiant intensity into relation (23), we find that

$$(fw)^m \exp(-f^2 w^2/2) \approx 0 \qquad \text{unless } |\mathbf{f}| \ll k. \tag{24}$$

Since the magnitude of the left-hand side of relation (24) is appreciable only when $f \leqslant 1/w$, condition (24) is equivalent to the condition

$$1/w^2 \ll k^2. \tag{25}$$

In physical terms, this inequality implies that a characteristic diffraction angle of a rather coherent beam, $\theta_d \sim \lambda/w$, must be small ($\theta_d \ll 1$); it characterizes the paraxial regime for fully coherent beams.

Next, we focus on the case of a nearly incoherent source ($\xi \to 1$). In this case, the right-hand side of Eq. (21) can be simplified by utilizing the asymptotic expression for the modified Bessel function for large values of its argument ($x \gg 1$), viz.,

$$I_m(x) \simeq \frac{1}{\sqrt{2\pi x}} \exp(x). \tag{26}$$

On substituting the resulting expression into relation (23) and after some algebra, one can cast condition (23) for the radiant intensity into the form

$$\frac{1}{f^2 w^2} \exp\left[-\frac{1-\xi}{(1+\sqrt{\xi})^2} f^2 w^2\right] \approx 0 \qquad \text{unless } |\mathbf{f}| \ll k. \tag{27}$$

Condition (27) is satisfied for wave numbers f such that $f^2 \leqslant (1+\sqrt{\xi})^2/w^2(1-\xi)$. Hence the condition for generation of a paraxial beam can be written in terms of the spatial coherence length σ_c and the wave number $k = 2\pi/\lambda$ in the form

$$\frac{(1+\sqrt{\xi})^2}{2\sqrt{\xi}} \frac{1}{{\sigma_c}^2} \ll k^2. \tag{28}$$

This condition implies that for a nearly incoherent source ($w \gg \sigma_c$), the diffraction angle of a beam is specified by the spatial coherence length, so that $\theta_d \sim \lambda/\sigma_c$, and this angle is small in the paraxial domain ($\theta_d \ll 1$).

4. PARAXIAL PROPAGATION OF THE BEAM AND THE M^2 FACTOR

The cross-spectral density of a beam at a pair of points $(\boldsymbol{\rho}, z)$ and $(\boldsymbol{\rho}', z')$ in the half-space $z > 0$ is related, in the paraxial domain, to the cross-spectral density at a pair of points in the source plane through the double Fresnel transform (Ref. 29, Chap. 10) (see also Ref. 30):

$$W(\boldsymbol{\rho}, \boldsymbol{\rho}', z, z') = \frac{k^2}{4\pi^2 z z'} \exp[ik(z-z')] \int \mathrm{d}^2\rho_1 \times \int \mathrm{d}^2\rho_2\, W(\boldsymbol{\rho}, \boldsymbol{\rho}', 0) \exp\left[\frac{ik}{2z}(\boldsymbol{\rho}-\boldsymbol{\rho}_1)^2\right] \times \exp\left[-\frac{ik}{2z'}(\boldsymbol{\rho}'-\boldsymbol{\rho}_2)^2\right]. \tag{29}$$

On substituting the mode expansion (2) into Eq. (29), we obtain for the cross-spectral density the expression

$$W(\boldsymbol{\rho}, \boldsymbol{\rho}', z, z') = \sum_{nm} \lambda_{nm} \psi_n^{m*}(\boldsymbol{\rho}, z) \psi_n^m(\boldsymbol{\rho}', z'), \tag{30}$$

where $\psi_n^m(\boldsymbol{\rho}, z)$ is the Fresnel transform of the source mode $\psi_n^m(\boldsymbol{\rho}, 0)$. The functional form of the Laguerre–Gauss source mode is well known at any point z from the theory of laser resonator modes.[24] One has

$$\psi_n^m(\boldsymbol{\rho}, z) = \left(\frac{w}{w_z}\right)\left(\frac{\sqrt{2}\rho}{w_z}\right)^m L_n^m\left(\frac{2\rho^2}{w^2}\right) \times \exp\left(-\frac{\rho^2}{w^2}\right)\exp(-im\phi) \times \exp\left\{i\left[kz-(m+1)\Phi_z+\frac{k\rho^2}{2R_z}\right]\right\}, \tag{31}$$

where w_z, R_z, and Φ_z are defined by the expressions

$$w_z = \left(w^2 + \frac{4z^2}{k^2 w^2}\right)^2, \tag{32}$$

$$R_z = z + \frac{k^2 w^4}{4z}, \tag{33}$$

$$\Phi_z = \arctan\left(\frac{2z}{kw^2}\right). \tag{34}$$

Further, on substituting Eq. (31) into the mode expansion (30) and using Eq. (5) to perform the summation, one arrives at the following expression for the cross-spectral density at an arbitrary pair of points:

$$W(\boldsymbol{\rho}, \boldsymbol{\rho}', z, z') = \frac{A\xi^{-m/2}}{1-\xi}\left(\frac{w^2}{w_z w_{z'}}\right)\exp[im(\phi-\phi')] \times \exp\{i[k(z-z')-(m+1)(\Phi_z-\Phi_{z'})]\} \times \exp[i(k\rho^2/2R_z - k\rho'^2/2R_{z'})] \times \exp\left[-\frac{1+\xi}{1-\xi}\left(\frac{\rho^2}{{w_z}^2}+\frac{\rho'^2}{{w_{z'}}^2}\right)\right] I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho\rho'}{w_z w_{z'}}\right). \tag{35}$$

It is seen from Eq. (35) that if one chooses any point, say (ρ', z'), to be a reference point, the overall phase of the cross-spectral density of the partially coherent beam rela-

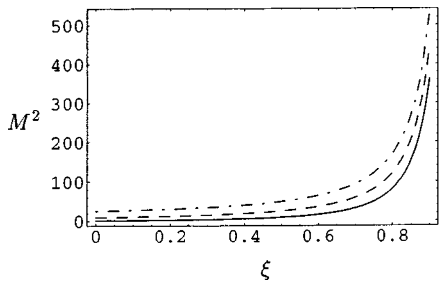

Fig. 5. Dependence of the M^2 beam factor on the coherence parameter ξ. The solid curve corresponds to a vortex-free beam with the angular index $m = 0$, whereas the dashed and dotted–dashed curves correspond to beams with topological charges $m = 2$ and $m = 4$, respectively.

tive to this point will have the same helicoidal shape as that of the phase of the fully coherent Laguerre–Gauss beam of Eq. (31). Therefore the wave front of the former beam is endowed with a vortex structure similar to that of the latter, with the azimuthal mode index m being a topological charge of the optical vortex.[15–18]

To describe more fully the quality of partially coherent beams, one may use invariant quality parameters introduced in Ref. 31 as early as 1985 (see also Ref. 32 for the rotationally symmetric case). However, it is sufficient for our purposes to calculate the beam quality factor of Siegman, M^2. For any beam with an axially symmetric intensity distribution, this factor is defined as[33]

$$M^2 = 2\pi\sigma_\rho\sigma_\infty, \tag{36}$$

where σ_ρ is the second-order moment of the spectral intensity distribution and σ_∞ is the second-order moment of the radiant intensity distribution. These moments are defined by the expressions

$$\sigma_\rho = \frac{\int \mathrm{d}^2\rho\, \rho^2 S(\boldsymbol{\rho})}{\int \mathrm{d}^2\rho\, S(\boldsymbol{\rho})}, \tag{37}$$

$$\sigma_\infty = \frac{\int \mathrm{d}^2(k\mathbf{s}_\perp)(k\mathbf{s}_\perp)^2 J(k\mathbf{s}_\perp)}{\int \mathrm{d}^2(k\mathbf{s}_\perp) J(k\mathbf{s}_\perp)}, \tag{38}$$

respectively. In our case, the spectral intensity $S(\boldsymbol{\rho})$ is given by Eq. (9), and the radiant intensity $J(\hat{\mathbf{s}})$ is expressed explicitly by Eq. (22). In this case, analytical expressions for the integrals in Eqs. (37) and (38) involve cumbersome combinations of hypergeometric functions. Therefore all the integrations have been performed numerically. The results are presented in Fig. 5. In this figure, the quality factor is plotted as a function of the parameter ξ for the vortex-free beam with the angular index $m = 0$ together with that of the beams carrying vortices with topological charges $m = 2$ and $m = 4$, respectively. It is seen from the figure that the presence of a vortex degrades the quality of the beam. This conclusion agrees well with earlier findings of Ref. 34, where it was shown that in the case of a generic vortex, in order to preserve a hole, which the vortex burns inside a beam, the vortex phase must counterbalance the diffraction. The competition between these effects deteriorates beam quality. It can also be seen from Fig. 5 that the quality factor increases approximately linearly with the increase of the parameter ξ (or, equivalently, with the decrease of the spectral coherence length) and remains relatively small until ξ reaches a value of approximately 0.55. The subsequent decrease of the spectral coherence length beyond this point results in a fast deterioration of beam quality.

5. CONCLUDING REMARKS

This analysis can be summarized by saying that a new family of partially coherent beams with a separable phase has been introduced. This phase acquires a vortex structure on paraxial propagation of the beam in free space. Any member of the family is generated by the incoherent superposition of the fully coherent, normalized, Laguerre–Gauss modes of arbitrary order, with the same azimuthal mode index. In complete analogy with fully coherent beams carrying optical vortices, the azimuthal mode index of the partially coherent beams that were introduced plays the role of a topological charge.[18] It was also shown that the cross-spectral density of the new class of beams is invariant under the spatial Fourier transformation in a plane transverse to the direction of propagation of the beam. This property, together with their shape invariance on paraxial propagation, makes these beams similar to ordinary Gaussian Schell-model (GSM) beams or to twisted Gaussian Schell-model (TGSM) beams. However, unlike the spectral degree of coherence of GSM and TGSM beams, the spectral degree of coherence of the new beams is *independent* of the relative orientation of a pair of points in the transverse plane. This remarkable property might be useful for applications where highly *isotropic* coherence properties of light are required.

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Address correspondence to Sergey A. Ponomarenko at the location on the title page or by e-mail, ponos @pas.rochester.edu.

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