Ultrashort pulse coherence properties in coherent linear amplifiers

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We examine coherence properties of small-area, intrinsically stationary statistical pulses propagating in amplifying media in the vicinity of an optical resonance. Any such medium acts as a coherent linear amplifier, amplifying and reshaping the pulse. We show that an initially nearly incoherent Gaussian Schell-model pulse becomes almost fully coherent and its state of coherence becomes nearly uniform across the temporal profile as the pulse propagates into the amplifying medium.

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1. INTRODUCTION

The interest in statistical properties of ultrashort pulses stems, in part, from the fundamental limitations noise imposes on the performance and accuracy of state-of-art fiber-optical communication systems [1]. To date, there has been extensive research on modeling statistical properties of intrinsically stationary [2,3] and cyclostationary [4,5] random pulses and pulse trains as well as on such fundamental issues as defining and measuring statistical pulse spectra [6,7] and cross-spectral correlations [7]. Space–time correlation dynamics of statistical pulses were also addressed, both theoretically [8,9] and experimentally [10], and several theories dealing with various representations of statistical pulses were advanced [11–13]. Further, statistical properties of pulses during their propagation in optical fibers [14,15] and generic linear [16–18] and nonlinear [19] dispersive media far from any optical resonances were examined.

At the same time, near-resonant propagation of statistical pulses in linear media has also been explored [20–23]. In particular, shape-invariant fully coherent pulses propagating in coherent linear amplifiers and absorbers in the resonant regime were discovered [20,21], and the influence of statistical properties on self-similar pulse evolution was examined in resonant absorber media [22]. Moreover, global correlation properties of generic partially coherent pulses in resonant linear absorbers were examined and the general area-correlation theorem was derived [23]. To our knowledge, however, coherence properties of ultrashort statistical pulses propagating in resonant amplifying media have not yet been studied.

In this work, we explore coherence properties of small-area, intrinsically stationary statistical pulses during their propagation in coherent amplifying media. First, we show that initially symmetric pulse intensity profiles become asymmetric upon propagation in the media. In particular, coherence properties of Gaussian Schell-model (GSM) pulses—which represent a rather generic model of intrinsically stationary pulses, generated, for instance, by temporal modulation of a statistically stationary source with a Gaussian spectrum [3]—become nearly uniform across the temporal profile of the pulse over sufficiently long propagation distances in the amplifier. This contrasts sharply with rapid variations of temporal coherence properties across the pulse profile for statistical pulses propagating in resonant linear absorbers [23]. We also note that coherence properties of GSM pulses remain uniform in conservative dispersive media, e.g., optical fibers [14]. Second, we demonstrate that even initially rather incoherent pulses become progressively more coherent upon propagation in amplifying media. Thus, not only do the media amplify and reshape the pulse, but they also reduce the noise associated with source fluctuations. This circumstance can have important implications for short-range optical communications with ultrashort pulses.

2. STATISTICAL PULSE PROPAGATION IN COHERENT LINEAR AMPLIFIERS

As the first step, we model the pulse propagation in a coherent resonant amplifier media. We describe a resonant medium using the two-level atom model. The pulse evolution in the medium can be described in the slowly varying envelope approximation by the reduced wave equation [20,24]

$$\partial_\zeta \Omega = i\kappa \langle \sigma \rangle_\Delta. \tag{1}$$

In Eq. (1), $\kappa = \omega N |d_{eg}|^2 / c\epsilon_0 \hbar$ is a coupling constant, with N being the atom density and d_{eg} being the dipole matrix element between the ground and excited states of an atom; $\Omega = 2 d_{eg} \mathcal{E}/\hbar$ is the complex pulse envelope amplitude in frequency units corresponding to the field envelope $\mathcal{E}$. In Eq. (1), $\langle \ldots \rangle_\Delta$ implies averaging over a distribution of detunings Δ of the carrier wave frequency ω_c from the atomic resonance frequency ω_0; as in our previous studies [23], we assume a Lorentzian detuning distribution with a characteristic decay time T_Δ associated with inhomogeneous broadening. The complex dipole envelope function σ and one-atom inversion w obey the Bloch equations in the form [24]

$$\partial_\tau \sigma = -(\gamma_\perp + i\Delta)\sigma - i\Omega w, \tag{2}$$

and

$$\partial_\tau w = -\gamma_\perp (w - w_{\rm eq}) - \frac{i}{2}(\Omega^* \sigma - \Omega \sigma^*). \tag{3}$$

Here $\gamma_\perp = 1/T_\perp$ is a dipole relaxation rate, and $T_\perp$ is a dipole relaxation time. In Eqs. (1)–(3) we introduced the shifted coordinates: $\zeta = z$ and $\tau = t - z/c$.

For small-area input pulses, one can neglect any amplifier gain depletion, implying that w can be well approximated by its equilibrium value $w_{\rm eq}$ (linear amplifier), i.e.,

$$w \approx w_{\rm eq} = 1. \tag{4}$$

The linearized dipole moment evolution equation then reads

$$\partial_\zeta \sigma = -(\gamma_\perp + i\Delta)\sigma - i\Omega. \tag{5}$$

Equations (1) and (5) can be solved using a Fourier-transform technique, resulting in

$$\mathcal{E}(\tau,\zeta) = \int_{-\infty}^{\infty} {\rm d}\omega \tilde{\mathcal{E}}_0(\omega) \exp\left[-i\omega\tau + \frac{\alpha\zeta}{2(1 - i\omega T_{\rm eff})}\right]. \tag{6}$$

Here $T_{\rm eff}^{-1} = T_\perp^{-1} + T_\Delta^{-1}$ is an effective relaxation rate that fully characterizes damping in the Lorentzian detuning distribution case. Also, $\alpha = 2\kappa/\gamma_\perp$ is a small-signal amplification coefficient. The incident pulse spectrum is given by

$$\tilde{\mathcal{E}}_0(\omega) = \int_{-\infty}^{\infty} \frac{{\rm d}t}{2\pi} \mathcal{E}(t,0) e^{i\omega t}. \tag{7}$$

The second-order coherence properties of small-area statistical pulses in the amplifying media are specified by the two-time correlation function $\Gamma(\tau_1,\tau_2,\zeta)$, defined as

$$\Gamma(\tau_1,\tau_2,\zeta) = \langle \mathcal{E}^*(\tau_1,\zeta)\mathcal{E}(\tau_2,\zeta)\rangle, \tag{8}$$

where the angular brackets denote ensemble averaging. It follows from Eqs. (6)–(8) that

$$\Gamma(\tau_1,\tau_2,\zeta) = \int_{-\infty}^{\infty} {\rm d}\omega_1 \int_{-\infty}^{\infty} {\rm d}\omega_2 \mathcal{W}_0(\omega_1,\omega_2) e^{i(\omega_1\tau_1 - \omega_2\tau_2)} \times \exp\left\{\alpha\zeta\left[\frac{1}{2(1 - i\omega_2 T_{\rm eff})} + \frac{1}{2(1 + i\omega_1 T_{\rm eff})}\right]\right\}. \tag{9}$$

Here $\mathcal{W}_0(\omega_1,\omega_2) = \langle \tilde{\mathcal{E}}_0^*(\omega_1)\tilde{\mathcal{E}}_0(\omega_2)\rangle$ is the cross-spectral density of the pulse fields at the source; it is related to the corresponding two-time correlation function, viz.,

$$\mathcal{W}_0(\omega_1,\omega_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{{\rm d}t_1{\rm d}t_2}{(2\pi)^2} e^{i(\omega_2 t_2 - \omega_1 t_1)} \Gamma_0(t_1,t_2). \tag{10}$$

3. INTENSITY AND COMPLEX DEGREE OF COHERENCE EVOLUTION FOR INTRINSICALLY STATISTICALLY STATIONARY PULSES IN COHERENT LINEAR AMPLIFIERS

Next, we explore the influence of the input pulse coherence on its subsequent propagation dynamics in the amplifier. To this end, we consider incident GSM pulses [2] as a rather generic statistical model of the source. The choice of the GSM model has two advantages. First, GSM pulses can be easily generated in a laboratory by "chopping" with a Gaussian temporal modulation function an output of a statistically stationary source with a Gaussian spectrum [3]. Second, being intimately related to statistically stationary sources, the generated pulses are intrinsically stationary with a Gaussian correlation function that decays very fast with the time difference. Thus, the conditions for the ergodicity hypothesis are met [25] and any time averages arising in actual experiments converge well to ensemble averages employed throughout this work.

The two-time correlation function of the GSM pulse can be written as

$$\Gamma_0(t_1,t_2) \propto \exp\left(-\frac{t_1^2 + t_2^2}{2t_p^2}\right) \exp\left[-\frac{(t_1 - t_2)^2}{2t_c^2}\right], \tag{11}$$

where t_p and t_c are the characteristic pulsewidth and coherence time, respectively. As we are interested in the coherent coupling between the near-resonant pulse and medium atoms, we focus on the short pulse limit such that $t_p \sim T_{\rm eff}$. In particular, we let $t_p = T_{\rm eff}/2$ throughout our numerical simulations, and introduce dimensionless variables, $Z = \alpha\zeta$ and $T = t/T_{\rm eff}$.

In Fig. 1, we display the behavior of the GSM pulse intensity profile, $I(T,Z) = \Gamma(T,T,Z)$, on propagation in the amplifier for two cases: (a) $t_c = 5t_p$, corresponding to an almost fully coherent input pulse, and (b) $t_c = t_p/5$, corresponding to a nearly incoherent input pulse. It is seen in Fig. 1 that as both pulses are amplified, their shapes become progressively more asymmetric, developing a long tail in the trailing edge. However, the amplifying medium affects the pulse with the shorter coherence time less than it does the more coherent pulse. This is because the less coherent pulse has a broader spectrum, containing a significant portion of its initial energy in the tails, outside the medium gain spectrum. Therefore, it is amplified less efficiently by the medium than is the more coherent pulse.

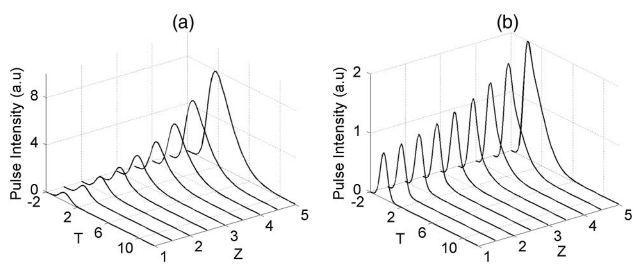

Fig. 1. Pulse intensity profile in arbitrary units (a.u) as a function of the dimensionless propagation distance Z for two partially coherent pulses: (a) $t_c = 5t_p$ and (b) $t_c = t_p/5$.

To drive this point home, we exhibit in Fig. 2 the energy gain factor, $G(Z) = W(Z)/W_0$, where the pulse energy W is

$$W(Z) \propto \int_{-\infty}^{\infty} \mathrm{d}T\Gamma(T, T, Z), \qquad (12)$$

for both pulses as a function of the propagation distance. It is clearly seen in the figure that the more coherent pulse is able to extract much more energy from the amplifying medium over the same propagation distance than is the less coherent pulse.

Next, we examine the behavior of the complex degree of coherence γ, defined (in dimensionless variables) as [25,26]

$$\gamma(T_1, T_2, Z) \equiv \frac{\Gamma(T_1, T_2, Z)}{\sqrt{I(T_1, Z)I(T_2, Z)}}. \qquad (13)$$

In Fig. 3 we display the complex degree of coherence of the pulse with $t_c = 5t_p$ for two propagation distances, $Z = 1$ and $Z = 5$. We can see from the figure that not only does the pulse become more coherent on propagation, but its coherence properties become more uniform as well. In the insets in Figs. 3(a) and 3(b), we display the corresponding pulse intensity at the same propagation distance. It is seen from Figs. 3(a) and 3(b) that the chosen ranges for T_1 and T_2 in both cases correspond to the time intervals within which most of the pulse energy resides at a given propagation distance. Thus, our γ plots are representative of the whole pulse.

It is interesting to compare the behavior of the fairly coherent pulse we have just studied with that of a relatively incoherent pulse with $t_c = t_p/5$. The evolution of the complex degree of coherence for the latter is shown in Fig. 4. Similarly to Fig. 3, we display the pulse intensity in the corresponding inset, which sets the T_1 and T_2 ranges. We can infer by comparing Figs. 3 and 4 that although qualitatively the behavior of γ is the same regardless of the initial state of pulse coherence, the coherence state of initially nearly incoherent pulses becomes nearly uniform across their temporal profile—apart from the pulse tails—and the magnitude of γ tends to unity with the propagation distance.

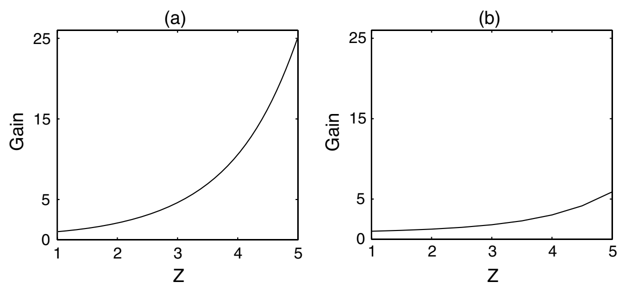

Fig. 2. Energy gain factor as a function of the dimensionless propagation distance Z for two partially coherent pulses: (a) $t_c = 5t_p$ and (b) $t_c = t_p/5$.

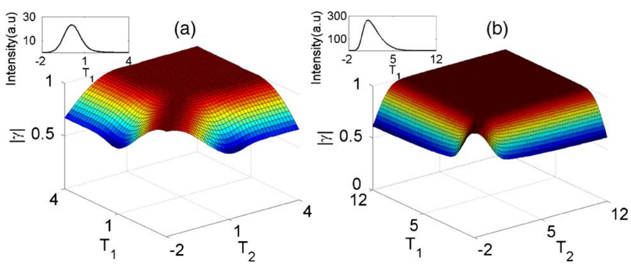

Fig. 3. (Color online) Magnitude of the complex degree of coherence of a short GSM pulse with $t_c = 5t_p$ for (a) $Z = 1$ and (b) $Z = 5$. Insets: the corresponding pulse intensity profiles.

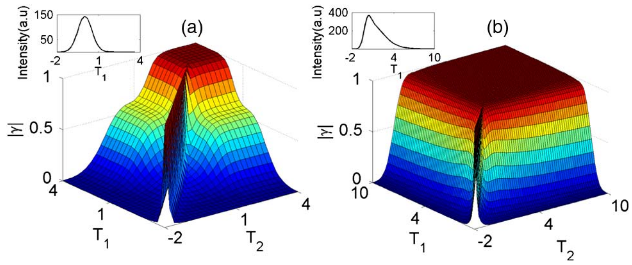

Fig. 4. (Color online) Magnitude of the temporal degree of coherence of a short GSM pulse with $t_c = t_p/5$ for (a) $Z = 1$ and (b) $Z = 5$. Insets: the corresponding pulse intensity profiles.

4. SUMMARY AND DISCUSSION

Finally, we mention that the small-area approximation, which is necessary for the system to be in the linear amplification regime, imposes severe constraints on the input pulse parameters and/or amplifier lengths. Hereafter we assume, for simplicity, the input pulse to be fully coherent, which is fine for the order-of-magnitude estimates. First of all, the pulse area $\mathcal{A}$ at the exit of the amplifier must be much smaller than π for nonlinear saturation effects to be negligible over the entire amplifier length [24]. Let us take $\mathcal{A} \sim 0.1$, say, implying that for the amplifier of length $L = 5L_B$, measured in Beer's absorption lengths, $L_B = \alpha^{-1}$, the input pulse area must be tiny, $\mathcal{A}_0 \sim 5 \times 10^{-4}$, where we used the area theorem for coherent amplifiers [27].

However, the very small magnitude of the input pulse area begs the question as to whether the incident pulse contains enough photons to be treated classically. To address this question, we estimate the input Gaussian pulse energy density,

$$w_0 = \frac{1}{2}\epsilon_0 c \int_{-\infty}^{\infty} \mathrm{d}t|\mathcal{E}(t, 0)|^2 = \frac{1}{2}\epsilon_0 c\sqrt{\pi}t_p\mathcal{E}_0^2, \qquad (14)$$

where $\mathcal{E}_0$ is the peak amplitude of the pulse. The input area can be estimated as

$$\mathcal{A}_0 = \frac{2d_{\mathrm{eg}}}{\hbar}\int_{-\infty}^{\infty} \mathrm{d}t\mathcal{E}(t, 0) = \frac{2d_{\mathrm{eg}}\sqrt{2\pi}t_p\mathcal{E}_0}{\hbar}. \qquad (15)$$

Eliminating the peak pulse amplitude from Eqs. (14) and (15), we arrive at the input energy density

$$w_0 = \frac{\epsilon_0 c\hbar^2\mathcal{A}_0^2}{16\sqrt{\pi}d_{\mathrm{eg}}t_p}. \qquad (16)$$

Assuming a picosecond laser pulse of 1 cm^2 cross section and taking $d_{\mathrm{eg}} \simeq 10^{-29}$ cm, which is appropriate for atomic vapors [24], we substitute these values into Eq. (16) to estimate the input pulse energy as $W_0 \sim 5 \times 10^{-13}$ J. It follows that the number of photons carried by the pulse can be estimated as $\mathcal{N}_0 = W_0/\hbar\omega_c \simeq 5 \times 10^6 \gg 1$, which is sufficiently large to treat the pulse as a classical electromagnetic field. However, this criterion can be easily violated as the amplifier length increases.

In summary, we studied partially coherent pulse propagation in resonant linear amplifiers, focusing on the change in pulse coherence properties upon propagation. We have shown that regardless of the initial state of pulse coherence, it becomes progressively more coherent during propagation in the amplifying medium. Moreover, the state of coherence becomes progressively more uniform across the temporal profile as the pulse propagates into the medium. We also discussed the constraints imposed by the linear amplification regime on the input parameters of the pulse and the amplifier length.

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