# Self-imaging of partially coherent light in graded-index media

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We demonstrate that partially coherent light beams of arbitrary intensity and spectral degree of coherence profiles can self-image in linear graded-index media. The results can be applicable to imaging with noisy spatial or temporal light sources. © 2015 Optical Society of America

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Imaging has been a prominent subject in optics and optical engineering for quite some time. Optical imaging is typically classified as coherent or incoherent depending on the coherence properties of a light source [1]. The term incoherent imaging is usually employed in situations where light sources are quite noisy so that the resulting electromagnetic fields possess random phases. Hence, one can describe the imaging process only in terms of appropriate second-order correlation functions of the (partially coherent) fields [2].

Although it is customary to employ optical instruments such as lenses, microscopes, telescopes etc. to ensure image formation [3], under certain conditions the properties of optical fields are conducive to light self-imaging even in the absence of any devices. In particular, transverse periodicity of coherent optical fields at the source gives rise to longitudinal periodicity on free-space propagation of the fields, which constitutes the Talbot effect [4]. Ever since its discovery, the Talbot effect has generated a flurry of research activity, both fundamental and applied, in the field of coherent optics [5–13]. Whenever optical sources are partially coherent, Talbot self-imaging can also occur for periodic source cross-spectral density functions or tensors, depending on whether the sources are completely or partially polarized [14–20]. In addition, partially coherent sources may have aperiodic intensity profiles and periodic coherence properties as, for instance, do recently introduced optical coherence lattices [21]. Although Talbot self-imaging is precluded for such sources, they can be attractive for free-space optical communications due to the discovered phenomenon of periodicity reciprocity: optical coherence lattices give rise to periodic angular-intensity distributions in the far zone concurrently with losing their coherence periodicity on free-space propagation [22].

We stress that, in general, the source intensity as well as the spectral degree of coherence periodicity are required for self-imaging. It will then be instructive to inquire whether self-imaging can be realized in an optical medium without any conditions on the source field and without relying on any imaging devices. We point out in this context that Talbot imaging of certain kinds of fully [23] and partially [24,25] spatially coherent sources in linear graded-index (GRIN) media were previously discussed.

In this Letter, we address this issue by showing that perfect self-imaging of any source field, regardless of its periodicity and its state of coherence, is indeed possible in a GRIN medium at certain distances away from the source. Our results can find applications in imaging with partially coherent spatial sources in free space or bulk optical media or with noisy pulses in optical fibers.

We consider a planar paraxial source, assuming it to be one-dimensional, for simplicity. We can then explore the generated beam propagation in a GRIN slab with the refractive index

$$n(x) = n_0 - n_1 x^2, \tag{1}$$

where $n_0$ is a refractive index on an axis of the system and $n_1$ ($n_1 \ll n_0$) characterizes the rate of refractive index variation in a plane transverse to the axis. In this work, we assume that the GRIN medium is focusing with $n_1 > 0$. We mention in passing that linear GRIN variation has also been shown to play an important role in the nonlinear regime of beam/pulse propagation by qualitatively altering such fundamental aspects as spatiotemporal field collapse [26,27] and soliton and self-similar nonlinear wave propagation in defocusing nonlinear GRIN media [28–30].

Hereafter, it will prove advantageous to introduce the following dimensionless variables

$$X = x/\sigma \quad \text{and} \quad Z = z/L_{\mathrm{D}}. \tag{2}$$

Here

$$\sigma = n_1^{-1/2}, \qquad L_{\mathrm{D}} = k/n_1, \tag{3}$$

where $k = \omega/c$ is a wave number corresponding to the carrier frequency $\omega$ of the source. We are now in a position to make our key assertion.

**Proposition.** *A beam of arbitrary-intensity profile and state of coherence can undergo self-imaging in a linear graded-index medium; perfect source images are formed at the distances that are multiple integers of* $2\pi L_{\mathrm{D}}$.

**Proof.** We begin by recalling that the cross-spectral density function of **any** beam ensemble at a pair of points $X_1$ and $X_2$ in any transverse plane $Z = \mathrm{const} > 0$ can be expressed in terms of the complex Gaussian representation (CGR) as [31]

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$$W(X_1, X_2, Z) = \int \mathrm{d}^2\alpha\, \mathcal{P}(\alpha)\psi_\alpha^*(X_1, Z)\psi_\alpha(X_2, Z). \quad (4)$$

Here $\alpha = \mathrm{Re}\alpha + i\,\mathrm{Im}\,\alpha$ is a complex variable, and $d^2\alpha \equiv d(\mathrm{Re}\,\alpha)d(\mathrm{Im}\,\alpha)$, $\mathcal{P}(\alpha)$ is a nonnegative distribution function to ensure non-negative definiteness of $W$ [2,31].

Each complex Gaussian mode (CGM) field at the source can be expressed as

$$\psi_\alpha(X, 0) = \frac{e^{-(\mathrm{Im}\,\alpha)^2}}{\pi^{1/4}} \exp\left[-\frac{(X - \sqrt{2}\alpha)^2}{2}\right]. \quad (5)$$

The CGMs are normalized,

$$\int \mathrm{d}X\, |\psi_\alpha(X, 0)|^2 = 1, \quad (6)$$

and form an overcomplete set such that

$$\int \mathrm{d}^2\alpha\, \psi_\alpha^*(X_1, 0)\psi_\alpha(X_2, 0) = \delta(X_1 - X_2). \quad (7)$$

Each CGM field obeys the paraxial-wave equation written in dimensionless variables as

$$\left(i\partial_Z + \frac{1}{2}\partial_X^2 - \frac{1}{2}X^2\right)\psi_\alpha(X, Z) = 0. \quad (8)$$

We will now employ a mathematical equivalence between the CGM mode propagation in a GRIN medium and a quantum harmonic oscillator evolution. To this end, let us observe that the CGM field can be thought of as the coordinate representation of a ket vector $|\alpha, Z\rangle$ in the Hilbert space such that $\psi_\alpha(X, Z) = \langle X|\alpha, Z\rangle$; the source CGM is just a coherent state of a quantum harmonic oscillator, $|\alpha, 0\rangle \equiv |\alpha\rangle$, as was shown in [31]. The mode propagation equation can then be cast into the form of the Schrödinger equation for $|\alpha, Z\rangle$ as

$$(i\partial_Z - \hat{H})|\alpha, Z\rangle = 0, \quad (9)$$

where the propagation distance $Z$ plays the role of time, and $\hat{H} = -\frac{1}{2}\partial_X^2 + \frac{1}{2}X^2$ is an analog of the quantum harmonic oscillator Hamiltonian. Proceeding with our quantum-oscillator analogy, we introduce the eigenstates of $\hat{H}$, viz.,

$$\hat{H}|n\rangle = \left(n + \frac{1}{2}\right)|n\rangle, \quad (10)$$

where $n$ is an integer.

As is well known [32], the coherent state can be expanded in a complete set of the oscillator eigenstates as

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle. \quad (11)$$

By formally integrating Eq. (9), we obtain the CGM propagation law in the form

$$|\alpha, Z\rangle = e^{-i\hat{H}Z}|\alpha\rangle. \quad (12)$$

It then follows from Eqs. (10)–(12) that the CGM evolution with the propagation distance is governed by

$$|\alpha, Z\rangle = e^{-iZ/2}|\alpha e^{-iZ}\rangle, \quad (13)$$

or, projecting back into the coordinate representation, we arrive at

$$\psi_\alpha(X, Z) = \pi^{-1/4}e^{-iZ/2}\exp\{-[\mathrm{Im}(\alpha e^{-iZ})]^2\} \times \exp\left[-\frac{(X - \sqrt{2}\alpha e^{-iZ})^2}{2}\right]. \quad (14)$$

It can be inferred from Eq. (14) that each CGM maintains its overall shape, but its amplitude is scaled by $e^{-iZ/2}$ and its complex displacement oscillates periodically with propagation distance $\alpha(Z) = \alpha e^{-iZ}$. It then follows from a quick analysis of Eqs. (4) and (14) that the cross-spectral density of any source is self-imaged over multiple integers of the distance $z_{\mathrm{SI}} = 2\pi L_{\mathrm{D}}$ in dimensional variables. This completes our proof.

As an example, let us consider a Gaussian Schell-model (GSM) source with a cross-spectral density of the form

$$W(X_1, X_2, 0) = I_0 e^{-(X_1^2+X_2^2)/2}\exp\left[-\frac{(X_1 - X_2)^2}{2\xi_c^2}\right]. \quad (15)$$

Here $I_0$ is a peak intensity of the source, $\xi_c = \sigma_c/\sigma$, where $\sigma_c$ is a transverse coherence length of the source, and we assumed, for simplicity, that the rms source width $\sigma_p$ is equal to the characteristic GRIN scale $\sigma_p = \sigma$. Introducing the notation,

$$\alpha = \frac{1}{\sqrt{2}}(u + iv), \quad (16)$$

the CGR weight distribution in this case is known as [31]

$$\mathcal{P}(\alpha) \equiv \mathcal{P}(u, v) = I_0\xi_c\delta(u)e^{-\xi_c^2 v^2/2}. \quad (17)$$

It then follows by substituting from Eqs. (14)–(17) into Eq. (4) after some straightforward algebra that the cross-spectral density of the beam in a transverse plane $Z = \mathrm{const} \geq 0$ is given by

$$W(X_1, X_2, Z) = \frac{I_0\xi_c}{\sqrt{\xi_c^2 + 2\sin^2 Z}}\exp\left(-\frac{X_1^2 + X_2^2}{2}\right) \times \exp\left[-\frac{(X_1 e^{iZ} - X_2 e^{-iZ})^2}{2(\xi_c^2 + 2\sin^2 Z)}\right]. \quad (18)$$

We can infer by comparing Eqs. (15) and (18) that at $Z = \pi m$, $m = 0, \pm 1, \pm 2, \ldots$, the GSM source self-images by replicating its cross-spectral density. We note that in a more general case, $\sigma_p \neq \sigma$, both the intensity and spectral degree of coherence [33] of a generated GSM beam change with the propagation distance inside GRIN media. However, it can be shown explicitly that, in accordance with our proposition, the source cross-spectral density periodically replicates itself.

We remark that although we have so far limited ourselves to a $(1+1)$D spatial case, the extension to two transverse dimensions is straightforward. The GRIN medium refractive index can then be generalized to

$$n(x, y) = n_0 - n_1(x^2 + y^2). \tag{19}$$

Because of the CGM-evolution-equation separability in such a medium in the Cartesian coordinates, we can factorize the source cross-spectral density as we did for optical coherence lattices [21,22] and prove self-imaging can happen, literally following our line of argument for each factor.

More instructively, though, the proven self-imaging can occur for pulses of any degree of temporal coherence propagating in optical fibers. The GRIN effect can be realized in the temporal domain by utilizing the quadratic electro-optical effect in a Kerr cell inside a fiber and varying a low-frequency voltage across the cell linearly in time [34]. As a result, the refractive index will vary quadratically in time. The straightforward extension of a familiar space–time analogy between partially coherent beam propagation in free space and partially coherent pulse propagation in homogeneous, linear fibers [35] to the GRIN case will ensure self-imaging of noisy pulses as well. Thus, our findings can contribute to the thriving field of temporal imaging in fiber optics [36].

In summary, we have shown that self-imaging can occur for spatial or temporal sources of arbitrary-intensity distribution and state of coherence in GRIN linear media or noninstantaneous fibers with a quadratic time response. Perfect self-images of the second-order-correlation properties take place at distances that are multiple integers of a characteristic medium period. Our results shall be useful for spatial imaging with partially coherent light in waveguides or bulk media or for temporal imaging with fluctuating pulses in optical fibers.

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