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Coherent pseudo-mode decomposition of a new partially coherent source class

SHUNCHUAN YANG, SERGEY A. PONOMARENKO,* AND ZHIZHANG (DAVID) CHEN

Department of Electrical and Computer Engineering, Dalhousie University, Halifax, Nova Scotia B3J 2X4, Canada
**Corresponding author: serpo@dal.ca*

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We introduce a new class of Schell-model temporal sources that admit a closed-form analytical decomposition in terms of coherent pseudo-modes. We explore the source mode and the corresponding eigenvalue structure and demonstrate that the lowest-order mode profile reflects the distribution of the source intensity as a function of time. We also examine the global degree of coherence of the class and show that it is virtually independent of the pulse intensity profile particulars at the source.

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The coherent mode decomposition (CMD) of the optical coherence theory has served as a versatile tool for analysis and synthesis of partially spatially or temporarily coherent light sources [1]. The second-order coherence properties of a pulse ensemble $\{V(t)\}$ generated by such a source are characterized by the mutual coherence function of the ensemble, defined as $\Gamma(t_1, t_2) = \langle V^*(t_1)V(t_2)\rangle$, where the angle brackets denote ensemble averaging. Within the CMD framework, the mutual coherence function can be represented as [1]

$$\Gamma(t_1, t_2) = \sum_n \lambda_n \psi_n^*(t_1)\psi_n(t_2). \quad (1)$$

Provided the modes are orthonormal,

$$\int_{-\infty}^{\infty} dt \psi_n^*(t)\psi_m(t) = \delta_{mn}, \quad (2)$$

the mode set $\{\psi_n(t)\}$ and the corresponding eigenvalues $\{\lambda_n\}$ can be determined by solving the homogeneous Fredholm integral equation:

$$\int_{-\infty}^{\infty} dt_1 \Gamma(t_1, t_2)\psi_n(t_1) = \lambda_n \psi_n(t_2). \quad (3)$$

Unfortunately, solving Eq. (3) presents, in general, a formidable mathematical task [2]. As a result, there has been only a limited number of sources for which coherent modes were determined. Apart from the classic case of Gaussian Schell-model sources [3,4], twisted Gaussian Schell-model sources [5,6], Bessel-correlated [7], modified-Bessel-correlated sources generating partially coherent vortex fields [8], and dark/anti-dark sources [9], which are propagation-invariant [10], comprise all known sources admitting a closed-form coherent mode decomposition. Some of these or related partially coherent sources were experimentally realized [11,12]

The introduction of an alternative to CMD representations for partially coherent sources [13,14] has precipitated the discovery of a multitude of new sources, including the so-called non-uniformly correlated sources [15], multi-Gaussian [16], cosine-Gaussian [17,18] and sinc [19] Schell-model sources, optical coherence gratings [20], and lattices [20,21], as well as other sources [22,23]. Instructively, it was recognized by the authors of [24] and [14] that the representations [13] and [14], respectively, can be equivalent to the CMD with nonorthogonal pseudo-modes. To the best of our knowledge, however, with the exception of optical coherence gratings and lattices, no realistic partially coherent sources amenable to the CMD in terms of pseudo-modes have been discussed to date.

In this Letter, we introduce a new class of Schell-model temporal sources which admit a closed-form analytical expansion in terms of pseudo-modes. We explicitly derive the pseudo-modes and the corresponding eigenvalues and show that the lowest-order mode mimics the temporal profile of the source, while the eigenvalue distribution determines global coherence properties of the source.

We start by focusing on the class of Schell-model sources with the mutual coherence function in the form

$$\Gamma(T_1, T_2) = \sqrt{I_1(T_1)I_2(T_2)}\left\{\frac{2J_1[\alpha_c(T_1 - T_2)]}{\alpha_c(T_1 - T_2)}\right\}, \quad (4)$$

where $I(T) \equiv \Gamma(T, T)$ is an arbitrary intensity profile of the source and $J_1(x)$ is a Bessel function of the first kind and first order. In Eq. (4), we introduced dimensionless variables as $T = t/\tau_I$ and $\alpha_c = \beta\tau_I$, where τ_I is a temporal width of the pulse and β is a characteristic inverse coherence time. In the dimensionless variables, the new sources are completely specified by the dimensionless coherence parameter α_c. The sought CMD of the source can be written as

$$\Gamma(T_1, T_2) = \sum_m \lambda_m \phi_m^*(T_1)\phi_m(T_2). \quad (5)$$

We relax the mode orthogonality condition, given by Eq. (2) in the expansion (5), but require mode normalization, i.e.,

$$\int_{-\infty}^{\infty} \mathrm{d}T |\phi_m(T)|^2 = 1. \tag{6}$$

Equations (5) and (6) imply that each eigenvalue specifies the amount of energy carried by the corresponding mode.

We stress that the lack of mode orthogonality precludes a systematic mode determination because the integral equation, Eq. (3), can no longer be derived. Thus, the modes can, in general, be determined only by inspection. To this end, we consider the following Bessel function identity [25]:

$$\frac{2J_1(x-y)}{x-y} = \sum_{m=1}^{\infty} 4m^2 \chi_m(x)\chi_m(y), \tag{7}$$

where

$$\chi_m(s) = J_m(s)/s. \tag{8}$$

It follows by inspection from Eqs. (5)–(8) that each normalized pseudo-mode can be expressed as

$$\phi_m(T) = \frac{\sqrt{I(T)}\chi_m(T)}{\left[\int_{-\infty}^{\infty} \mathrm{d}T I(T)\chi_m^2(T)\right]^{1/2}}, \tag{9}$$

and the corresponding eigenvalue reads

$$\lambda_m = 4m^2 \int_{-\infty}^{\infty} \mathrm{d}T I(T)\chi_m^2(T). \tag{10}$$

Since the right-hand side of Eq. (10) is manifestly non-negative, $\lambda_m \geq 0$ for any m, thereby ensuring non-negative definiteness of the mutual coherence function (4). We notice in passing that the presented pseudo-mode CMD works for any pulse intensity profile of the source which makes the novel sources similar to previously discovered Bessel-correlated spatial sources. However, the CMD of the latter involves orthogonal modes because of the angular mode function orthogonality [7].

As the modes and eigenvalues explicitly depend on the source intensity profile, we must specify the latter to proceed further. Let us consider the following pulse intensity at the source:

$$I(T) \propto \exp[-T^{2q}]. \tag{11}$$

Here $q \geq 1$ is a positive integer. The case $q = 1$ corresponds to a Gaussian source, describing generic laser pulses, whereas the case $q > 1$ leads to a super Gaussian source which models well the pulses generated by many mode-locked lasers [26]. Using Eqs. (8)–(11), we can numerically evaluate the modes.

In Figs. 1–3, we display the first five mode patterns. It is seen in the figures that the lowest-order mode (mode 1) profile flattens as q increases. Another interesting phenomenon, observed when comparing the mode patterns of Gaussian and various super Gaussian source pulses in Figs. 1–3, is manifested by the leading and trailing edge sharpening of the lowest-order pseudo-mode as the source pulse profile switches from the Gaussian to a progressively more and more super Gaussian one. It implies that the lowest-order mode actually determines, at least qualitatively, the overall source pulse shape for both Gaussian and super Gaussian pulse sources. Higher-order modes are seen to be time-delayed and their pattern complexity grows as the mode index increases. The larger the mode index, the more it time-delayed. On the other hand, the peak amplitude of a super Gaussian source mode increases with the mode index as well. Additionally, higher-order mode profiles of super Gaussian pulses are observed to merge together in the regions away from their peaks or nodes. In addition, all pseudo-modes exhibit a particular kind of symmetry, either a point or an axial one.

The novel source coherence properties are statistically stationary, implying that it is of the Schell-model type. Such sources can be readily generated from statistically stationary sources with the same degree of coherence by using, for instance, electro-optical or acousto-optical time modulators [27]. We can rearrange the pseudo-mode eigenvalues in the decreasing order, $\lambda_1 \geq \lambda_2 \geq \lambda_3 \ldots$, as is seen from numerical simulations. Indeed, we display in Figs. 4 and 5 the eigenvalues

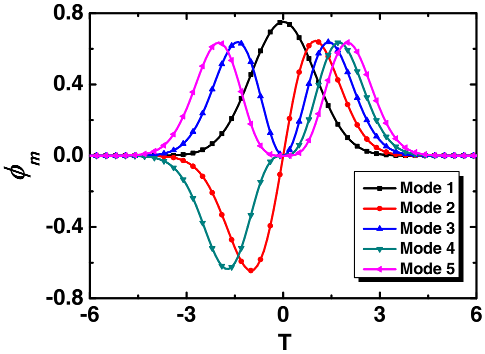

Fig. 1. First five modes of a Gaussian source as functions of time T for $\alpha_c = 1$.

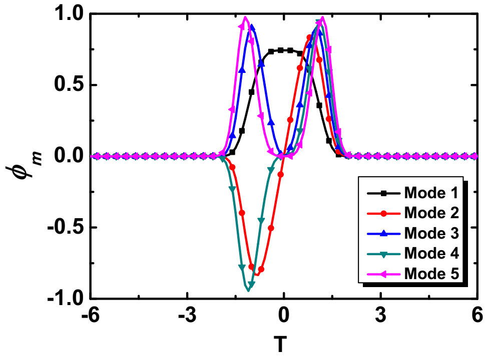

Fig. 2. First five modes of a super Gaussian source with $q = 2$ as functions of T for $\alpha_c = 1$.

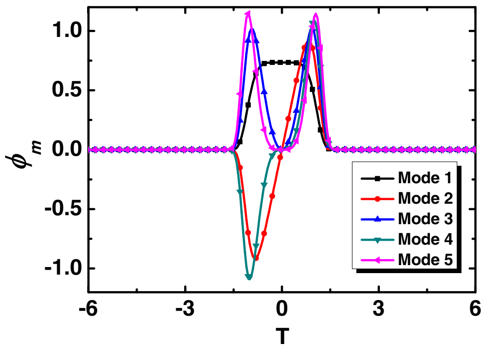

Fig. 3. First five modes of a super Gaussian source with $q = 3$ as functions of T for $\alpha_c = 1$.

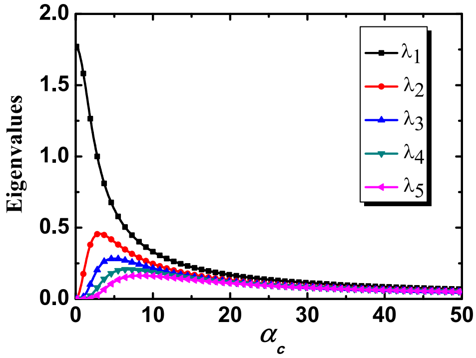

Fig. 4. Gaussian source eigenvalues λ_m versus α_c.

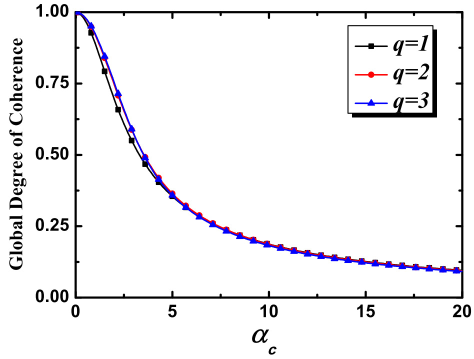

Fig. 6. Global degree of coherence of the source as a function of α_c.

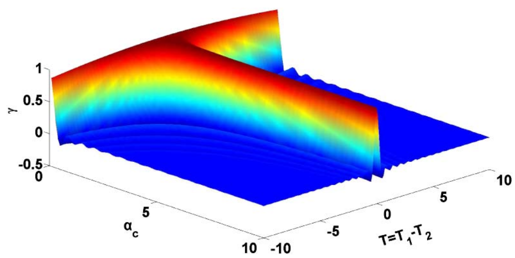

Fig. 7. Local degree of coherence of the source as a function of the time difference and source coherence parameter α_c.

corresponding to the first five modes as functions of the coherence parameter α_c. It can be inferred from the figures that the lowest-order mode eigenvalue has the largest magnitude. As α_c increases, the source coherence is reduced, resulting in a monotonous decrease of the lowest-order eigenvalue. The eigenvalues associated with higher-order modes attain maxima at certain values of α_c, followed by a monotonous decrease with α_c. In the limit of a completely incoherent source, all eigenvalues tend to the same limit, implying energy equipartition among the modes (thermal-like distribution).

The global degree of coherence, defined as the fraction of the pulse energy carried by the most energetic, lowest-order mode, can be expressed as [28]

$$\nu = \frac{\lambda_1}{\sum_{m=1}^{\infty} \lambda_m}. \quad \text{(12)}$$

In Fig. 6, we exhibit the source global degree of coherence which monotonously decreases with α_c. It follows that the global degree of coherence diminishes as the source becomes less coherent, as expected. We also notice that the global degree of coherence is only weakly affected by the magnitude of q. Thus, the overall source coherence properties are affected marginally by a specific shape of its intensity profile.

For completeness, we can also calculate the local degree of coherence of the source at two time instances T_1 and T_2, defined as $\gamma(T_1, T_2) = \Gamma(T_1, T_2)/\sqrt{I(T_1)I(T_2)}$ [1,29]. We present the results in Fig. 7 as a function of the time difference, $T_1 - T_2$, and the source coherence parameter α_c.

In conclusion, we derived a closed-form analytical decomposition of novel Schell-model partially coherent temporal light sources in terms of nonorthogonal pseudo-modes. We examined the pseudo-mode and the associated eigenvalue properties and showed that the lowest-order pseudo-mode captures the qualitative behavior of the source pulse intensity profile. We also determined the global degree of coherence of the source and elucidated its dependence on the source coherence parameter and its intensity profile. The fact that the global degree of coherence turns out to be virtually independent of the source intensity distribution, the feature the new sources share with the well-known Gaussian Schell-model sources [1], hints to the universality of the feature for any Schell-model type source. The conjectured universality trait should not be confused with universal features of the local degree of coherence, previously discovered in the context of a certain class of statistically homogeneous light sources [30]. We stress, that although our results were formulated for temporal sources, they also hold for one-dimensional spatial sources; the extension to two-dimensional sources with Cartesian symmetry is straightforward because of the cross-spectral density separability in the Cartesian coordinates. Finally, we believe that the obtained results advance our, rather incomplete at present, understanding of the pseudo-mode decompositions of partially coherent light sources, composed of nonorthogonal modes.

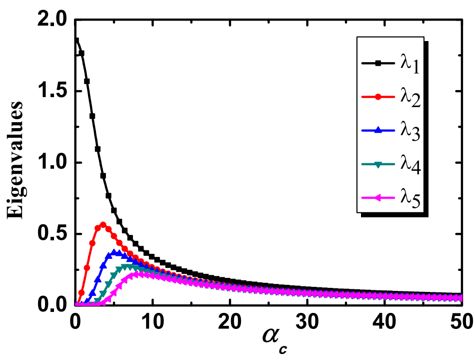

Fig. 5. Super Gaussian source eigenvalues λ_m versus α_c; the source intensity parameter $q = 3$.

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