Universal structure of field correlations within a fluctuating medium

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We study the structure of the second-order correlation function of scalar wave fields, which are generated by statistically stationary sources, fluctuating within a homogeneous dissipative medium. We derive a closed-form analytical expression for the spectral degree of coherence of the wave field. If the dissipation in the medium is sufficiently small, and the source fluctuations are statistically isotropic, the degree of spatial coherence of the field produced by any such source is shown to be proportional to the imaginary part of the Green's function of the system, with a proportionality factor depending on the dimensionality of the field. The result holds for wave fields in both three and two spatial dimensions, but not in one dimension. We discuss the physical nature of such universal forms of the spectral degree of coherence.

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I. INTRODUCTION

Wave fields, which are generated within a homogeneous medium by a statistically stationary fluctuating source, are ubiquitous in physics. In electrodynamics, they are associated with the electromagnetic field generated in a medium by the fluctuating polarization [1,2]. Similar systems are encountered in acoustics, in connection with sound waves produced by the fluctuating density of a medium. It should be mentioned that the dimensionality of the space of the wave field need not be limited to three. Thus, electromagnetic waves, which propagate on a surface and decay exponentially in the direction normal to this surface [3], hydrodynamic waves on the surface of a fluid, and cylindrical acoustic waves [4] serve as examples of two-dimensional wave fields which can be excited by surface currents or mechanical forces, for example. A vibrating string [5] and a transmission line [6] are driven one-dimensional wave fields.

Since so many different physical systems can be described within the framework of a single model, it is instructive to study general statistical properties of such a model. A natural question arises regarding the relation between the correlation functions of the wave fields and those of the fluctuating sources that generate these fields. The properties of the second-order correlation function of a field are of major importance for the understanding of many statistical phenomena in condensed matter theory [7]. The structure of the second-order correlation function of scalar wave fields produced by statistically stationary sources in a three-dimensional homogeneous medium was considered in Ref. [8]. In that paper, it was demonstrated that, provided the dissipation in the medium is negligible, the spectral degree of coherence of the wave field has the same functional form regardless of the properties of the source, provided its fluctuations are statistically isotropic. This surprising result was further discussed in Ref. [9]. However, the approach of Ref. [8] relies heavily on some particular properties of the three-dimensional Green's function of the system in configuration space. This feature of the method makes it difficult to draw conclusions about the *universal* properties of the spectral degree of coherence of wave fields that are shared by wave fields of *different* spatial dimensionality. Another open question is the influence of dissipation in the medium on the nature of the field correlations.

In the present paper, we address these questions by considering the second-order correlations of the wave fields generated by statistically stationary sources, which fluctuate in a D-dimensional ($D=1$, 2, or 3), unbounded, homogeneous, dissipative medium. We derive a general analytical expression for the spectral degree of coherence of the wave field in terms of the cross-spectral density of the fluctuating source. We then show that if (1) the medium is nonabsorbing, (2) the source fluctuations are statistically isotropic, and (3) the dimensionality of the space is two and three, the spectral degree of coherence of such a wave field is given by a *universal* function, which is proportional to the imaginary part of the Green's function of the system. We briefly discuss why such universality does not exist for one-dimensional wave fields nor for any wave fields in the presence of significant dissipation in the medium. Unlike the well-known expressions for the second-order correlations of electromagnetic fields due to thermal fluctuations [2,10], our results hold for any linear wave field, which may be far from thermal equilibrium.

II. GENERAL EXPRESSION FOR THE SPECTRAL DEGREE OF COHERENCE OF THE WAVE FIELD

Consider a statistical ensemble $\{U(\mathbf{r},\omega)e^{-i\omega t}\}$ of strictly monochromatic, scalar wave fields [11,12] all at the same frequency ω. The propagation of the field is governed by the inhomogeneous wave equation

$$\left[\nabla^2+n^2(\omega)\frac{\omega^2}{c^2}\right]U(\mathbf{r},\omega)=-4\pi\rho(\mathbf{r},\omega). \tag{1}$$

Here, $\{\rho(\mathbf{r},\omega)\}$ is the ensemble of fluctuating source density, that produce such a field, and $n(\omega)$ is the (generally complex) refractive index of the medium [13]. The second-order statistical properties of the source and of the wave field may be specified by the cross-spectral density functions

$$W_\rho(\mathbf{r}_1,\mathbf{r}_2,\omega)=\langle\rho^*(\mathbf{r}_1,\omega)\rho(\mathbf{r}_2,\omega)\rangle \tag{2}$$

and

$$W_U(\mathbf{r}_1,\mathbf{r}_2,\omega)=\langle U^*(\mathbf{r}_1,\omega)U(\mathbf{r}_2,\omega)\rangle, \tag{3}$$

where the angular brackets denote the average over the statistical ensembles of realizations of the source and of the field. We introduce the spectral degree of coherence of the wave field by the expression [14]

$$\mu_U(\mathbf{r}_1,\mathbf{r}_2,\omega)=\frac{W_U(\mathbf{r}_1,\mathbf{r}_2,\omega)}{\sqrt{W_U(\mathbf{r}_1,\mathbf{r}_1,\omega)}\sqrt{W_U(\mathbf{r}_2,\mathbf{r}_2,\omega)}}. \tag{4}$$

Because of the statistical homogeneity of the source fluctuations, it is convenient to work in Fourier space. Let

$$\tilde{U}(\mathbf{k},\omega)=\int d^Dr\, U(\mathbf{r},\omega)e^{-i\mathbf{k}\cdot\mathbf{r}}, \tag{5}$$

and

$$\tilde{\rho}(\mathbf{k},\omega)=\int d^Dr\, \rho(\mathbf{r},\omega)e^{-i\mathbf{k}\cdot\mathbf{r}} \tag{6}$$

be the D-dimensional Fourier transforms of the wave field and of the source density, respectively. It follows at once from Eq. (1) and from definitions (5) and (6) that the Fourier transforms of the wave field $U(\mathbf{r},\omega)$ and of the source density $\rho(\mathbf{r},\omega)$ are related by the expression

$$\tilde{U}(\mathbf{k},\omega)=\frac{4\pi\tilde{\rho}(\mathbf{k},\omega)}{k^2-n^2(\omega)\omega^2/c^2}. \tag{7}$$

The statistical homogeneity of the source, which we have assumed, implies that

$$W_\rho(\mathbf{r}_1,\mathbf{r}_2,\omega)\equiv W_\rho(\mathbf{r},\omega), \tag{8}$$

where $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$. On introducing the $2D$-dimensional Fourier transform of the cross-spectral density $\tilde{W}_\rho(\mathbf{k}_1,\mathbf{k}_2,\omega)$ of the fluctuating source by the expression

$$W_\rho(\mathbf{r}_1,\mathbf{r}_2,\omega)=\int\frac{d^Dk_1}{(2\pi)^D}\int\frac{d^Dk_2}{(2\pi)^D}e^{i(\mathbf{k}_1\cdot\mathbf{r}_1+\mathbf{k}_2\cdot\mathbf{r}_2)}\times\tilde{W}_\rho(\mathbf{k}_1,\mathbf{k}_2,\omega), \tag{9}$$

condition (8) is readily seen to be equivalent to the relation

$$\tilde{W}_\rho(\mathbf{k}_1,-\mathbf{k}_2,\omega)=(2\pi)^D\delta(\mathbf{k}_1-\mathbf{k}_2)\langle|\tilde{\rho}(\mathbf{k}_1,\omega)|^2\rangle, \tag{10}$$

where $\delta(\mathbf{k})$ is the D-dimensional Dirac delta function.

On substituting from Eq. (7) into Eq. (3) and taking the ensemble average with the aid of Eqs. (2) and (10) and also making use of Eq. (9), we obtain for the cross-spectral density of the wave field the expression

$$W_U(\mathbf{r},\omega)=(4\pi)^2\int d^DR\, W_\rho(\mathbf{R})\times\int\frac{d^Dk}{(2\pi)^D}\frac{e^{i\mathbf{k}\cdot(\mathbf{R}-\mathbf{r})}}{|k^2-n^2(\omega)\omega^2/c^2|^2}. \tag{11}$$

It follows at once from Eq. (8) and from the definition (4) of the spectral degree of coherence of the field that

$$\mu_U(\mathbf{r},\omega)=\frac{W_U(\mathbf{r},\omega)}{W_U(0,\omega)}. \tag{12}$$

The integral in Eq. (11) can be evaluated explicitly by a straightforward though somewhat tedious calculation outlined in the Appendix. The result is

$$\mu_U^{(D)}(\mathbf{r},\omega)=\frac{\int d^DR\, W_\rho(\mathbf{R})g_D(\mathbf{R}-\mathbf{r})}{\int d^DR\, W_\rho(\mathbf{R})g_D(\mathbf{R})}, \tag{13}$$

where the function $g_D(\mathbf{x})$ characterizing the propagation properties of the medium is

$$g_D(\mathbf{x})=\begin{cases}\mathrm{sinc}(k|\mathbf{x}|)e^{-\gamma|\mathbf{x}|} & \text{in three dimensions}\\ H_0^{(1)}(\zeta|\mathbf{x}|)-H_0^{(1)}(-\zeta^*|\mathbf{x}|) & \text{in two dimensions}\\ \cos(k|\mathbf{x}|-\phi/2)e^{-\gamma|\mathbf{x}|} & \text{in one dimension.}\end{cases} \tag{14}$$

Here,

$$k(\omega)=\omega\epsilon'(\omega)/c|\epsilon(\omega)|^{1/2} \tag{15a}$$

and

$$\gamma(\omega)=\omega\epsilon''(\omega)/c|\epsilon(\omega)|^{1/2} \tag{15b}$$

are the real and imaginary parts, respectively, of the complex wave number $\zeta(\omega)=k(\omega)+i\gamma(\omega)$, associated with the frequency ω. Further, $\epsilon(\omega)=\epsilon'(\omega)+i\epsilon''(\omega)$ is the complex dielectric constant at frequency ω. The generally complex refractive index $n(\omega)=\sqrt{\epsilon(\omega)}$. The "phase lag" $\phi(\omega)$ is defined by the expression $\phi(\omega)=\arctan[\epsilon''(\omega)/\epsilon'(\omega)]$. In Eq. (14), $\mathrm{sinc}(x)\equiv\sin(x)/x$ [15], and $H_0^{(1)}(x)$ is the Hankel function of the first kind and of zero order.

Expressions (13) and (14) provide a closed-form representation for the spectral degree of coherence of the wave field in terms of the cross-spectral density of a statistically homogeneous source. We emphasize that unlike the method of Ref. [8], which permits the treatment of only three-dimensional wave fields without absorption, our approach makes it possible to study wave fields in dissipative media of any spatial dimensionality. However, a detailed analysis of Eqs. (13) and (14), reveals that if, dissipation in the medium is non-negligible, the correlation properties of the wave fields do, in general, depend on those of the driving sources, even if the sources are statistically isotropic. By a *statisti-*

cally isotropic source we mean, of course, a source whose cross-spectral density depends on $\mathbf{r}_1$ and $\mathbf{r}_2$ only through the distance $|\mathbf{r}_1-\mathbf{r}_2|$, ie.,

$$W_\rho(\mathbf{r}_1,\mathbf{r}_2,\omega)\equiv W_\rho(|\mathbf{r}_1-\mathbf{r}_2|,\omega). \tag{16}$$

In order to gain some insight into the influence of dissipation on the universality of the field correlations, we may note that in the presence of appreciable dissipation (absorption) the spectral degree of coherence of the field at any pair of points either can be a monotonic function of the distance between the points, or can decrease with increasing separation between the points in an oscillatory manner. Suppose first that the cross-spectral density of the source is a monotonic function, which decreases with increasing distance between the points. Suppose, further, that at every frequency ω, the typical inverse absorption length $\gamma(\omega)$ is much greater than the typical wave number $k(\omega)$, i.e., $\gamma(\omega)\gg k(\omega)$, so that, as is seen from Eq. (14), $g_D(\mathbf{x})$ is also a monotonically decreasing function of the separation between the points. It then follows at once from Eq. (13) that the spectral degree of coherence of the wave field will not display an oscillatory behavior. On the other hand, if $W_\rho(|\mathbf{x}|)$ decreases with increasing distance between the points in an oscillatory manner, and the inverse absorption length $\gamma(\omega)$ is of the same order of magnitude as $k(\omega)$, it follows from Eq. (13) that the spectral degree of coherence of the wave field may exhibit spatial oscillations. This heuristic argument shows why the presence of absorption leads to a different functional form of the spectral degree of coherence of the field, depending on a particular relation among the parameters characterizing propagation of the field in the medium.

III. UNIVERSAL FORM OF THE SPECTRAL DEGREE OF COHERENCE OF THE FIELD GENERATED BY A STATISTICALLY ISOTROPIC SOURCE

We will now study the case when the dissipation in the medium is very weak, i.e, when $\gamma\ll k$, and the fluctuating source is statistically isotropic. In this case, the general expression (13) reduces to

$$\mu_U^{(D)}(\mathbf{r},\omega)=\frac{\int_0^\infty dR\,R^{D-1}W_\rho(R)\int d\Omega_{\mathbf{R}}g_D(\mathbf{R}-\mathbf{r})}{\int_0^\infty dR\,R^{D-1}W_\rho(R)\int d\Omega_{\mathbf{R}}g_D(\mathbf{R})}, \tag{17}$$

and the function $g_D(\mathbf{x})$, defined by Eq. (14), becomes

$$g_D(\mathbf{x})=\begin{cases}\mathrm{sinc}(k|\mathbf{x}|) & \text{in three dimensions}\\ J_0(k|\mathbf{x}|) & \text{in two dimensions}\\ \cos(k|\mathbf{x}|) & \text{in one dimension.}\end{cases} \tag{18}$$

Let us first consider in more detail three-dimensional wave fields. In this case,

$$g_3(\mathbf{x})=4\pi\,\mathrm{Im}\,G_3(|\mathbf{x}|,\omega)=4\pi\,\mathrm{Im}\left(\frac{e^{ik|\mathbf{x}|}}{4\pi|\mathbf{x}|}\right), \tag{19}$$

where $G_3(\mathbf{r}_1,\mathbf{r}_2,\omega)$ is the Green's function of the 3D system. This Green's function can be represented by the series [cf. formula 8.531(1) of Ref. [16]]

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}=ik\sum_{l=0}^{\infty}j_l(kr_<)h_l^{(1)}(kr_>)\times\sum_{m=-l}^{l}Y_{lm}^*(\theta',\phi')Y_{lm}(\theta,\phi). \tag{20}$$

Here, $r_>$ and $r_<$ refer to the greater and the smaller of $|\mathbf{r}|$ and $|\mathbf{r}'|$, respectively; $j_l(x)$ and $h_l^{(1)}(x)$ are spherical Bessel and spherical Hankel functions of the first kind and of order l, and $Y_{lm}(\theta,\phi)$ are spherical harmonics. On substituting from Eq. (19) into Eq. (18) and using Eq. (20), we obtain for the spectral degree of coherence the expression

$$\mu_U^{(3)}(\mathbf{r}_1,\mathbf{r}_2,\omega)=\frac{\sin(k|\mathbf{r}_1-\mathbf{r}_2|)}{k|\mathbf{r}_1-\mathbf{r}_2|} \tag{21}$$

which agrees with the *universal* form of the field correlations derived by a different method in Ref. [8].

An advantage of our approach is that the two-dimensional case can be treated similarly. Indeed, it follows at once from Eqs. (18) that

$$g_2(\mathbf{x})=\mathrm{Re}[H_0^{(1)}(k|\mathbf{x}|)], \tag{22}$$

where $H_0^{(1)}(x)$ is the Hankel function of the first kind and zero order. We recall that the Green's function of the two-dimensional reduced wave equation is [17]

$$G_2(\mathbf{r}_1,\mathbf{r}_2,\omega)=-\frac{i}{4}H_0^{(1)}(k|\mathbf{r}_1-\mathbf{r}_2|). \tag{23}$$

The Hankel function $H_0^{(1)}(x)$ may be expanded in the series [cf. formula 8.531(2) of Ref. [16]]

$$H_0^{(1)}(k|\mathbf{r}-\mathbf{r}'|)=\sum_{m=-\infty}^{\infty}J_m(kr_<)\,H_m^{(1)}(kr_>)e^{im(\phi-\phi')}, \tag{24}$$

where $J_m(x)$ is the Bessel function of the first kind and of order m. On substituting from Eqs. (22) and (24) into Eqs. (17) and (18), one finds that the spectral degree of coherence of the two-dimensional wave field generated by a two-dimensional homogeneous, isotropic source has the simple form

$$\mu_U^{(2)}(\mathbf{r}_1,\mathbf{r}_2,\omega)=J_0(k|\mathbf{r}_1-\mathbf{r}_2|). \tag{25}$$

This result demonstrates that the field correlations produced by a statistically isotropic source in a space of two dimensions have also a universal form.

It is seen from Eqs. (21) and (25) that

$$\mu_U^{(D)}(\mathbf{r}_1,\mathbf{r}_2,\omega)=C_D\mathrm{Im}[G_D(\mathbf{r}_1,\mathbf{r}_2,\omega)], \tag{26}$$

where the numerical factor C_D is

$$C_D = \begin{cases} 4\pi/k & \text{in three dimensions} \\ 4 & \text{in two dimensions.} \end{cases} \quad (27)$$

Equation (26) demonstrates that the *structure* of field correlations within a fluctuating, statistically isotropic, nonabsorbing medium is determined entirely by the propagation properties of the medium, i.e, by its Green's function, and not by any particular source distribution. Hence, the universality of the field correlations is a property shared by all such volume and surface wave fields. We mention that our general treatment incorporates some well-known results as special cases: blackbody radiation [10,18] and the field correlations within a δ-correlated primary spherical source of radius that is much greater than the wavelength [19].

A detailed analysis of Eq. (17) together with Eq. (18) indicates that the spectral degree of coherence of one-dimensional wave fields does not have a universal functional form. This nonuniversality may appear surprising, but it can be understood by examining the implications of the definition (16) of a homogeneous, statistically isotropic source. Equation (16) implies that the cross-spectral density of the source is invariant with respect to (i) rotations in a D-dimensional space and (ii) inversion of $\mathbf{r}_1$ and $\mathbf{r}_2$ about the origin. Since both these operations are possible in three- and two-dimensional spaces, but not in one-dimensional space, where rotations do not exist, the statistical isotropy is a stringent enough condition to yield universality only for two- and three-dimensional systems.

IV. DISCUSSION AND SUMMARY

To understand better the results of the present paper, one should keep in mind the following circumstance. The universality of the field correlations, which we have explored, occurs only if the source fluctuations are statistically homogeneous and isotropic in a space of the *same* dimensionality as that of the generated wave field. For instance, the surface waves have universal correlation properties if the generating source is statistically isotropic in two-dimensional space. This situation can be compared with some other cases that were previously considered [20,21]. In particular, in Ref. [20], the correlations of the fields generated by two-dimensional, statistically homogeneous, Lambertian sources, radiating into the three-dimensional half space $z>0$, were studied. It has been demonstrated that, provided evanescent components of the field may be neglected, such a source is statistically isotropic in the source plane $z=0$. It can be readily shown, however, that the fields produced by such sources do *not* exhibit universal correlation properties. A similar conclusion holds for a one-dimensional Lambertian source radiating into the half space $z>0$, which was investigated in Ref. [21].

Another property shared by the universal forms of the spectral degree of coherence of volume and surface wave fields is worth noting. Equations (21) and (25) indicate that the spectral coherence length of the field correlations is *of the order* of the wavelength. On the other hand, emission of light into the half space $z>0$ by a 3D thermal source located within a homogeneous conducting medium that occupies the half space $z<0$, was considered in [22]. It was demonstrated in that paper that at certain frequencies the spectral coherence length of the field in the near zone may be much *smaller* than the wavelength of the radiation. The difference between this particular system and the class of systems that we have studied in this paper has its origin in the boundary conditions. The presence of the boundary $z=0$ not only affects the structure of the Green's function of the system considered in Ref. [22], but also breaks the statistical isotropy of the system. As a result the structure of the field correlations is no longer universal. The remarkably short spectral correlation length at a particular frequency found in [22] can be explained by an anomalously small Beer's absorption length at that frequency.

Correlation properties of the fields driven by stochastic sources have also been studied in connection with the theory of homogeneous hydrodynamic turbulence [23]. In particular, the two-point correlations of the velocity fields were shown to acquire certain universal properties in the so-called inertial range of spatial scales, where one can neglect the influence of viscosity. However, unlike the linear wave equation that we have discussed in this paper, the hydrodynamic equations are inherently nonlinear. The latter circumstance precludes a detailed analytical calculation of the second-order correlation function of the fields associated with homogeneous turbulence.

In summary, we have investigated in this paper the form of spectral cross-correlation functions of scalar wave fields generated by fluctuating sources in homogeneous dissipative media. We have found that there is a general form of the spectral degree of coherence of such wave fields, and we have shown that, when the wave field is generated by a homogeneous, statistically isotropic source in a transparent, homogeneous medium, the correlation properties of such a field are *independent* of the nature of the source distribution, and are determined by the imaginary part of the Green's function of the system. We also explained the breakdown of such a universality in one spatial dimension. Finally, only certain second-order correlation properties of homogeneous, statistically isotropic fields have been addressed in this paper. Whether universal properties also exist for higher-order field correlations is an open question.

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APPENDIX: EVALUATION OF THE INTEGRAL IN EQ. (11)

We consider the integral

$$I_D = \int \frac{d^D k}{(2\pi)^D} \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{|k^2 - n^2(\omega)\omega^2/c^2|^2}. \quad (A1)$$

Let us first study the case $D=3$. On performing the elementary angular integration, Eq. (A1) reduces to

$$I_3=\frac{1}{2\pi^2|\mathbf{x}|}\int_0^\infty dk\,\frac{k\sin(k|\mathbf{x}|)}{|k^2-n^2(\omega)\omega^2/c^2|^2}$$
$$=\frac{1}{(2\pi)^2 i|\mathbf{x}|}\int_{-\infty}^\infty dk\,\frac{k\,e^{ik|\mathbf{x}|}}{|k^2-n^2(\omega)\omega^2/c^2|^2}. \qquad (A2)$$

Since $e^{ik|\mathbf{x}|}$ is an analytic function of k in the upper half plane of the complex k plane, the integral in Eq. (A2) can be evaluated in the complex plane by closing the contour of integration in the upper half plane and applying the residue theorem. One readily finds that

$$I_3=\frac{\epsilon'}{4\pi\gamma|\epsilon|}\,\frac{\sin(k|\mathbf{x}|)}{k|\mathbf{x}|}e^{-\gamma|\mathbf{x}|}, \qquad (A3)$$

where, as before, ϵ' denotes the real part of the complex dielectric constant ϵ.

The evaluation of the corresponding integral in 2D is slightly more complicated. We first carry out the angular integration with the aid of the integral representation for Bessel functions [16] [formula 8.411], viz.,

$$J_0(x)=\int_0^{2\pi}\frac{d\phi}{2\pi}\exp(ix\cos\phi). \qquad (A4)$$

The integral in Eq. (A1), with $D=2$, then reduces to

$$I_2=\int_0^\infty\frac{dk}{2\pi}\,\frac{k\,J_0(k|\mathbf{x}|)}{|k^2-n^2(\omega)\omega^2/c^2|^2}. \qquad (A5)$$

Let us now recall the definitions of the Hankel functions of the first and second kinds,

$$H_0^{(1)}(z)=J_0(z)+iN_0(z), \qquad (A6a)$$

$$H_0^{(2)}(z)=J_0(z)-iN_0(z), \qquad (A6b)$$

where $J_0(z)$ and $N_0(z)$ are the Bessel and Neumann functions, respectively. We also make use of the following properties of Hankel functions (cf. Eqs. 9.1.39 of Ref. [24]):

$$H_\nu^{(2)}(z)=-e^{i\pi\nu}H_\nu^{(1)}(e^{i\pi}z), \qquad (A7a)$$

$$H_\nu^{(1)}(e^{i\pi}z)=-e^{-i\pi\nu}H_\nu^{(2)}(z). \qquad (A7b)$$

On substituting into Eq. (A5) from Eq. (A6) and using Eqs. (A7), one can express the integral in Eq. (A5) as

$$I_2=\int_{-\infty}^\infty\frac{dk}{4\pi}\,\frac{k\,H_0^{(1)}(k|\mathbf{x}|)}{|k^2-n^2(\omega)\omega^2/c^2|^2}. \qquad (A8)$$

Because $H_0^{(1)}(z)$ is analytic in z in the upper half of the complex z plane, one can close the contour of integration in that half plane and, on performing the integration, one obtains for I_2 the expression

$$I_2=\frac{\pi}{2\epsilon''\omega^2/c^2}[H_0^{(1)}(\zeta|\mathbf{x}|)-H_0^{(1)}(-\zeta^*|\mathbf{x}|)]. \qquad (A9)$$

Here ϵ'' stands, as before, for the imaginary part of the complex dielectric constant ϵ, and ζ is the complex wave number defined below Eqs. (15) of the text.

The integral for the one-dimensional case ($D=1$) can be evaluated directly with the help of the residue theorem. One finds that

$$I_1=\frac{e^{-\gamma|\mathbf{x}|}}{2(|\epsilon|)^{1/2}\gamma\omega^2/c^2}\cos(k|\mathbf{x}|-\phi/2), \qquad (A10)$$

where ϕ is the phase lag defined in the text below Eqs. (15). Finally, on combining Eqs. (A3), (A9), and (A10), we arrive at the expression (14) for $g_D(\mathbf{x})$.

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