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**SPECIAL TOPIC**

# Non-Gaussian statistics of partially coherent light in atmospheric turbulence*

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SPECIAL TOPIC — Optical field manipulation

# Non-Gaussian statistics of partially coherent light in atmospheric turbulence*

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We derive theoretically and verify experimentally a concise general expression for the normalized intensity correlations (IC) of partially coherent light in a weak atmospheric turbulence in the fast detector measurement regime. The derived relation reveals that the medium turbulence acts, in general, as an additional noise source enhancing the IC of partially coherent beams. The maximum of the beam IC is, in general, enhanced, causing the fields to exhibit super-Gaussian statistics. On the other hand, the relation indicates that turbulence-induced noise is negligible for sufficiently low coherence light, which reveals the condition for the turbulence-free correlation imaging.

**Keywords:** partially coherent beam, atmospheric turbulence, intensity correlation

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## 1. Introduction

The atmospheric turbulence effects on laser beam statistics have attracted considerable attention over the years because of widespread applications of such beams to free-space optical communications and remote sensing.[1,2] The random atmospheric fluctuations cause beam wander, intensity scintillations, and extra beam spreading beyond diffraction.[3] The intensity scintillations described by the normalized equal-point intensity–intensity correlation function represent the main reason behind the information quality degradation in communication protocols involving beam propagation through the turbulent atmosphere.[4–8] Yet, the scintillation effects themselves carry important information about the turbulent medium.[9]

At the same time, measuring the intensity correlations (ICs) of random light, one can recover the information hidden in partially coherent light fields or speckles through, e.g., ghost imaging protocols.[10–12] The phase aberrations and medium dispersion can be canceled with the help of certain IC measurement techniques,[13–15] which has potential application to aberration-free microscopy. Recently, Alves and co-authors showed that the image can be retrieved from an IC measurement of the speckles even if the speckles are partially blocked by amplitude obstacles.[16] In the above studies, the amplitude and phase aberrations were considered deterministic. However, a turbulence induces spatially and temporarily varying random phase fluctuations.

Some time ago Banakh and co-authors[17,18] thoroughly examined the IC of partially coherent light in the turbulent atmosphere. They showed that the IC measurement in the presence of turbulence can fall into one of the two categories: slow and fast detector regimes. In the slow detector regime, the detector response time $\tau_d$ is much longer than the characteristic coherence time $\tau_s$ of the optical fields, but much shorter than the characteristic time $\tau_a$ associated with the turbulence fluctuations. It follows that the detector is too sluggish to follow the field fluctuations, but it can detect the medium turbulence. In the fast detector regime, $\tau_d \ll \tau_s \ll \tau_a$, implying that the detector is capable of tracking both the intensity and medium fluctuations. In the slow detector case, Banakh and co-authors

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found that the scintillation strength of a light field propagating in the turbulent atmosphere, characterized by the IC magnitude, can be decreased by reducing the spatial coherence of the source.[17] This is a well established characteristic of partially coherent light propagation in the turbulent atmosphere by now.[1,2] In the fast detector case, the relative dispersion of intensity on the beam axis is known to always exceed unity under any turbulence conditions.[18]

In this paper, we examine both theoretically and experimentally the IC of a partially coherent light beam in a weak atmospheric turbulence as measured by a fast detector. We develop a concise general expression, for the first time to the best of our knowledge, for the partially coherent beam IC in the turbulent atmosphere. The derived relation indicates that the turbulence can be viewed as an extra noise source that, in general, boosts the optical field fluctuations. The relation, on the one hand, reveals non-Gaussian statistics of the light fields in turbulence, and, on the other hand, shows the possibility of turbulence-free correlation imaging similar to the modality reported in Refs. [10,19].

## 2. Theoretical analysis

We first consider a scalar, quasi-monochromatic, stochastic beam-like field propagating along the mean direction (say, $z$-axis) into the half-space ($z>0$). The second-order statistical properties of the field in the space-frequency domain are characterized by the two-point cross-spectral density function $W(\boldsymbol{r}_1,\boldsymbol{r}_2,z)=\langle E^*(\boldsymbol{r}_1,z)E(\boldsymbol{r}_2,z)\rangle_{\mathrm{s}}$, where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are two arbitrary spatial points in the transverse plane of the beam, $E(\boldsymbol{r},z)$ is a random electric field, the asterisk and angular brackets with the subscript $s$ denote complex conjugation and ensemble averaging over the field realizations, respectively. Hereafter, we drop the angular frequency $\omega$ for notational brevity. The two-point intensity–intensity correlation function $C(\boldsymbol{r}_1,\boldsymbol{r}_2,z)$ of the random field, characterizing the higher-order statistics of the field, is defined as[20]

$$C(\boldsymbol{r}_1,\boldsymbol{r}_2,z)=\langle E^*(\boldsymbol{r}_1,z)E(\boldsymbol{r}_1,z)E^*(\boldsymbol{r}_2,z)E(\boldsymbol{r}_2,z)\rangle_{\mathrm{s}}. \quad (1)$$

By assuming statistically stationary fields obeying Gaussian statistics, equation (1) may be expressed in terms of the second-order degree of coherence $|\mu(\boldsymbol{r}_1,\boldsymbol{r}_2,z)|$ by applying the Siegert theorem as $C(\boldsymbol{r}_1,\boldsymbol{r}_2,z)=I(\boldsymbol{r}_1,z)I(\boldsymbol{r}_2,z)[1+|\mu(\boldsymbol{r}_1,\boldsymbol{r}_2,z)|^2]$, where $I(\boldsymbol{r},z)=W(\boldsymbol{r},\boldsymbol{r},z)$ denotes the average intensity distribution. $|\mu(\boldsymbol{r}_1,\boldsymbol{r}_2,z)|$ is bounded between zero and unity. We define a normalized IC function $g(\boldsymbol{r}_1,\boldsymbol{r}_2,z)$ as

$$g(\boldsymbol{r}_1,\boldsymbol{r}_2,z)=\frac{C(\boldsymbol{r}_1,\boldsymbol{r}_2,z)}{I(\boldsymbol{r}_1,z)I(\boldsymbol{r}_2,z)}=1+|\mu(\boldsymbol{r}_1,\boldsymbol{r}_2,z)|^2. \quad (2)$$

It follows from Eq. (2) that $g(\boldsymbol{r}_1,\boldsymbol{r}_2,z)$ is bounded between 1 and 2 for the optical fields with Gaussian statistics. Monitoring laser beam intensity fluctuations with a slow detector on the beam propagation through the atmospheric turbulence has been well documented to date.[3,17] We stress, though, that the statistics in Eq. (2) can only be examined with a fast detector. A slow detector would yield the result $g(\boldsymbol{r}_1,\boldsymbol{r}_2,z)=1$, implying that any IC information carried by the random field itself would be completely lost.

The random field of a light beam transmitted through the atmospheric turbulence can be expressed, within the accuracy of the Rytov approximation, in a multiplicative form $E_{\mathrm{t}}(\boldsymbol{r},z)=E(\boldsymbol{r},z)\exp[\Psi(\boldsymbol{r},z)]$, where $\Psi(\boldsymbol{r},z)$ is a complex phase perturbation induced by the turbulence.[3] In the weak fluctuation regime, the Rytov variance of a plane wave is much less than unity, $\sigma_{\mathrm{R}}^2=1.23C_n^2k^{7/6}z^{11/6}\ll 1$, where $C_n^2$ is the turbulence structure constant and $k$ is the beam carrier wave number. It follows that the beam IC function can be expressed as

$$C_{\mathrm{t}}(\boldsymbol{r}_1,\boldsymbol{r}_2,z)=\langle E_{\mathrm{t}}^*(\boldsymbol{r}_1,z)E_{\mathrm{t}}(\boldsymbol{r}_1,z)E_{\mathrm{t}}^*(\boldsymbol{r}_2,z)E_{\mathrm{t}}(\boldsymbol{r}_2,z)\rangle_{\mathrm{s}}. \quad (3)$$

Substituting $E_{\mathrm{t}}(\boldsymbol{r},z)$ into Eq. (3) and assuming that the source and medium fluctuations are mutually independent, we obtain for the IC function the expression

$$C_{\mathrm{t}}(\boldsymbol{r}_1,\boldsymbol{r}_2,z)=C(\boldsymbol{r}_1,\boldsymbol{r}_2,z)F_4(\boldsymbol{r}_1,\boldsymbol{r}_2,z), \quad (4)$$

where

$$F_4(\boldsymbol{r}_1,\boldsymbol{r}_2,z)=\left\langle \mathrm{e}^{\Psi^*(\boldsymbol{r}_1,z)+\Psi(\boldsymbol{r}_1,z)+\Psi^*(\boldsymbol{r}_2,z)+\Psi(\boldsymbol{r}_2,z)}\right\rangle_{\mathrm{t}} \quad (5)$$

is the fourth-order correlation function of the complex phase perturbation. The angular brackets with the subscript t in Eq. (5) denote the ensemble average over the medium fluctuations. It follows that the normalized IC function of the partially coherent beam in turbulence can be written as

$$g_{\mathrm{t}}(\boldsymbol{r}_1,\boldsymbol{r}_2,z)=\left[1+|\mu(\boldsymbol{r}_1,\boldsymbol{r}_2,z)|^2\right][1+B_{\mathrm{I}}(\boldsymbol{r}_1,\boldsymbol{r}_2,z)], \quad (6)$$

with

$$B_{\mathrm{I}}(\boldsymbol{r}_1,\boldsymbol{r}_2,z)=\frac{F_4(\boldsymbol{r}_1,\boldsymbol{r}_2,z)}{F_2(\boldsymbol{r}_1,\boldsymbol{r}_1,z)F_2(\boldsymbol{r}_2,\boldsymbol{r}_2,z)}, \quad (7)$$

where $F_2(\boldsymbol{r}_1,\boldsymbol{r}_2,z)=\left\langle \mathrm{e}^{\Psi^*(\boldsymbol{r}_1,z)+\Psi(\boldsymbol{r}_2,z)}\right\rangle_{\mathrm{t}}$ is the second-order correlation function of the complex phase perturbation.

The rather elegant relation, Eq. (6), is the key result of this work. It implies that as measured by a fast detector, the atmospheric turbulence can be regarded as an extra noise source enhancing the intensity fluctuations of light by a factor of $1+B_{\mathrm{I}}(\boldsymbol{r}_1,\boldsymbol{r}_2,z)$. We then introduce a scintillation index of the beam in a turbulent medium, defined as $\sigma_{\mathrm{I}}^2(\boldsymbol{r},z)=B_{\mathrm{I}}(\boldsymbol{r}_1,\boldsymbol{r}_2,z)$. Next, recalling that for most realistic sources, the source correlations decay with the separation between any two points $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ such that $|\mu(\boldsymbol{r},\boldsymbol{r})|=1$, we can infer that $g_{\mathrm{t}}(\boldsymbol{r},z)=2[1+\sigma_{\mathrm{I}}^2(\boldsymbol{r},z)]$, i. e., the normalized IC maximum depends only on the beam scintillation index. It follows at once that $g_{\mathrm{t}}(\boldsymbol{r},z)\geq 2$, implying non-Gaussianity of the intensity fluctuation statistics.[20] We note that our conclusions regarding non-Gaussian statistics of the field in turbulence are consistent with the result of Banakh and Buldakov.[18]

At the same time, it follows from Eq. (6) that if $\sigma_{\rm I}^2(\boldsymbol{r},z)$ is small enough, one can fulfill the conditions required for turbulence-free correlation imaging.[10,19] Further, it is well known that partially coherent beams are less affected by the atmospheric turbulence than their coherent counterparts are: The smaller the spatial coherence width, the smaller the scintillation index.[17] Thus, the conditions for turbulence-free imaging can be realized by employing random light beams of very low spatial coherence. As an aside remark, we note that in the slow detector regime, equation (6) reduces to $g_{\rm t}(\boldsymbol{r},z) = 2[1+B_{\rm I}(\boldsymbol{r}_1,\boldsymbol{r}_2,z)]$. Thus, the correlation function $B_{\rm I}(\boldsymbol{r}_1,\boldsymbol{r}_2,z)$ of medium fluctuations can be recovered from slow detector measurements.

## 3. Experimental results

We now report our experiment results in the lab simulated atmospheric turbulence to verify our theoretical predictions for ICs of a Gaussian Schell-model (GSM) beam. We display an experimental setup for generating the GSM beam and measuring the normalized IC in the turbulent medium in Fig. 1. In part A of Fig. 1, we exhibit a setup for generating the GSM beam with a controllable spatial coherence width (further technical details can be found in Subsection 2.1 of Ref. [21]). We transmit thereby the generated GSM beam through a 60 cm long path within the thermally induced turbulence generated by an electric heating plane, followed by a spatial light modulator (SLM) loaded with random phase screens, modeling a random medium with the Kolmogorov turbulence spectrum. Finally, the beam arrives at a slow detector located in a transverse plane about 40 cm behind the SLM. We use a charge-coupled device (CCD) as the slow detector to measure the turbulence-induced fluctuations encapsulated in $B_{\rm I}(\boldsymbol{r}_1,\boldsymbol{r}_2,z)$. We then utilize a Hanbury Brown and Twiss type setup to implement a fast detector. As is shown in part C of Fig. 1, the beam is first split into two twin portions by a beam splitter (BS), with the transmitted fields captured by the single photon detectors $D_1$ and $D_2$. We then send the output detector signals to a coincidence circuit (CC) to measure the space–time distribution of the normalized IC function.

To verify the slow and fast detector approximations experimentally, we first study the behavior of the intensity fluctuations in the time domain, from which we can assess the characteristic time $\tau_{\rm s}$ of the partially coherent source itself and $\tau_{\rm a}$ of the intensity fluctuations caused by the turbulence. In this test, the CC with the resolution time of $\tau = 1.5625$ ns is used to measure a normalized temporal IC ($\boldsymbol{r}_1 = \boldsymbol{r}_2 = 0$) function of the field in the receiver plane, i.e., $g^{(2)}(\tau) = \langle I(\tau)I(0)\rangle / [\langle I(\tau)\rangle \langle I(0)\rangle]$. In Fig. 2(a) we present the experimental results (black solid curve) for the normalized temporal IC function of the GSM beam with the spatial coherence width $\delta_{\rm c} = 0.67$ mm in the receiver plane in the absence of turbulence. We conclude from a Gaussian fit, a red solid curve in Fig. 2(a) that the characteristic time $\tau_{\rm s}$ is around 0.3 µs. For a GSM beam with the spatial coherence width in the receiver plane ranging from 0.092 mm to 1.03 mm, the characteristic time $\tau_{\rm s}$ is in the range of 0.1 µs to 0.6 µs [not shown in Fig. 2(a)]. On the other hand, we display in Fig. 2(b) the normalized IC function of a fully coherent laser beam – the RGGD is now removed and hence no source fluctuations are present – in the receiver plane when turbulence is present. It follows from a numerical fit that the characteristic time $\tau_{\rm a}$ of the turbulence induced intensity fluctuations is about 30 ms. Therefore, the CCD with the integrated time $\tau_{\rm d} = 1$ ms can be viewed as the slow detector and the CC with its coincidence time $\tau_{\rm d} = 1.5625$ ns acts as the fast detector. In addition, one can see from Fig. 2(b) that the on-axis scintillation of a Gaussian beam in the lab-generated turbulence is about 0.25, i.e., $\sigma_{\rm I}^2 = g^{(2)}(\tau = 0) - 1 = 0.25$. If we consider the Gaussian beam as a plane wave for approximation, the estimated structure constant from the Rytov variance $1.23C_n^2 k^{7/6} z^{11/6}$ is about $C_n^2 = 3\times 10^{-9}$ m$^{-2/3}$. In the calculation, the wavelength of the beams is $\lambda = 632.8$ nm and the distance $z$ is chosen to be $z = 0.6$ m. The atmospheric coherence length is usually described by the Fried parameter $r_0$. Under the assumption of the plane wave and weak fluctuations, this parameter can be expressed as $r_0 = (0.42C_n^2 k^2 z)^{-3/5}$. By inserting the values of $C_n^2$, $k$, and $z$ into the expression of $r_0$, the Fried parameter is about 1.2 mm.

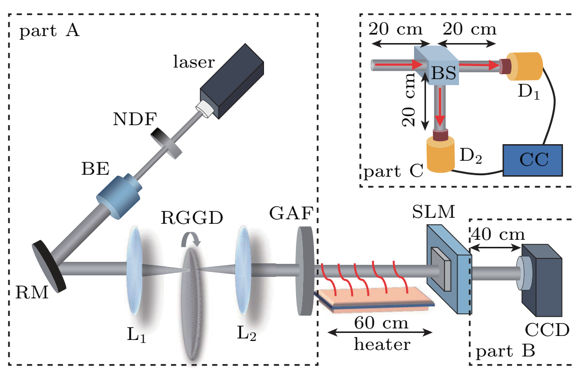


**Fig. 1.** Experimental setup for generating the GSM beam (part A) and measuring its intensity correlations in the lab-generated turbulence by using the slow (part B) and fast (part C) detectors. NDF: neutral density filter; BE: beam expander; RM: reflected mirror: $L_1$ and $L_2$: thin lenses; RGGD: rotating ground glass disk; GAF: Gaussian amplitude filter; SLM: spatial light modulator; BS: beam splitter; $D_1$ and $D_2$: single photon detectors; CC: coincidence circuit.

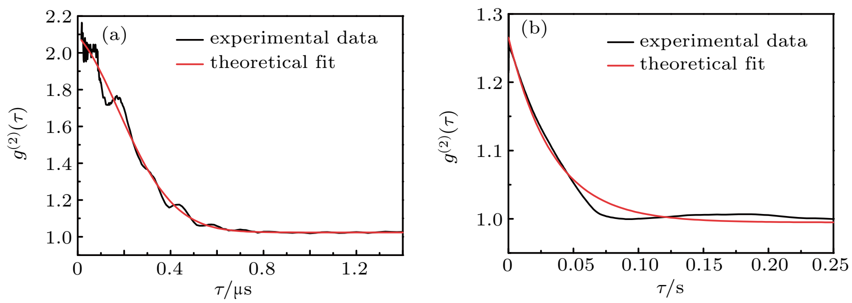


**Fig. 2.** Experimental results for the normalized temporal IC function of (a) GSM beam and (b) fully coherent Gaussian beam in the lab-generated turbulence as functions of time delay $\tau$.

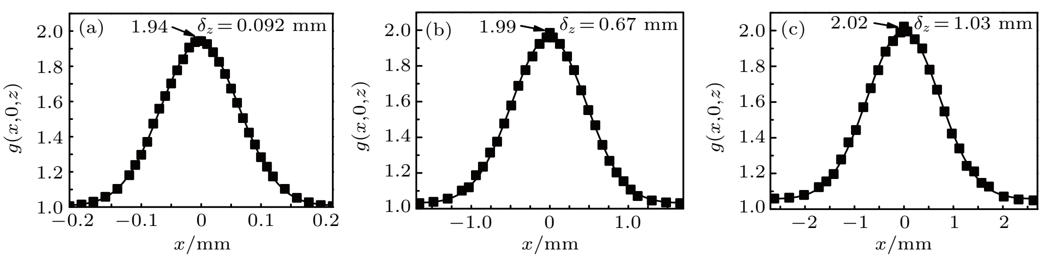


**Fig. 3.** Experimental results for the normalized IC function of the GSM beams with different spatial coherence widths in the receiver plane, inferred from fast detector measurements in the absence of turbulence. The solid curves show a corresponding Gaussian fit.

It follows from Eq. (6) that the first term on the l.h.s, characterizing light source fluctuations only, can be measured by the fast detector. In Fig. 3, we exhibit the experimental results for the normalized IC function $g(\boldsymbol{r}_1,\boldsymbol{r}_2,z)$ of the generated GSM beams of different spatial coherence widths in the receiver plane in the absence of turbulence. In the experiment, one single-photon detector was fixed at point (0,0) [$\boldsymbol{r}_2 = (0,0)$], while the other was scanning along the $x$-axis [$\boldsymbol{r}_1 = (x,0)$]. Our experimental data can be very well fit by a Gaussian curve, implying that the GSM beam statistics remained Gaussian after free-space propagation, as expected. We then obtain the spatial coherence widths in Figs. 3(a)–3(c) in the receiver plane to be 0.092 mm, 0.67 mm, and 1.03 mm, respectively, from the Gaussian fit.

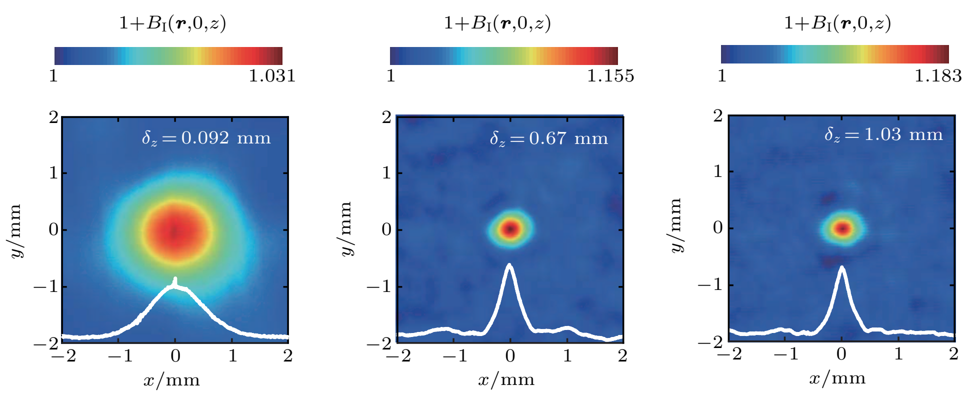


**Fig. 4.** Experimental results for $1+B_\mathrm{I}(\boldsymbol{r},0,z)$ of the GSM beams with different spatial coherence widths in the receiver plane tracked by a slow detector as the beams are transmitted through turbulence b. The white curves correspond to the cross-line at $y = 0$.

The second term on the l.h.s. of Eq. (6) is the normalized IC measured by a slow detector (CCD), which is essentially due to the medium fluctuations only. In Fig. 4, we show our experimental results for the normalized IC function detected by the CCD as the GSM source beams are transmitted through the lab-generated turbulence. The spatial coherence widths of the beams in Fig. 4 are the same as those in Fig. 3. We find that the width of the spatial distribution of $1+B_\mathrm{I}(\boldsymbol{r},0,z)$ in the transverse plane is closely related to the spatial coherence of the GSM beam. At the same time, the maximum-taking place at $\boldsymbol{r} = 0$ of $1+B_\mathrm{I}(\boldsymbol{r},0,z)$ increases with the spatial coherence. For the spatial coherence widths $\delta_\mathrm{c} = 0.092$ mm, 0.67 mm, and 1.03 mm, the maxima of $1+B_\mathrm{I}(\boldsymbol{r},0,z)$ are 1.031, 1.155, and 1.183, respectively. Overall, our results are consistent with those reported in Refs. [1,2], indicating that the partially coherent beams with low spatial coherence are more likely to resist turbulence-induced intensity scintillations.

Next, $g_t(\boldsymbol{r}_1, \boldsymbol{r}_2, z)$ of Eq. (6) can be tracked by the fast detector. The rectangles in Figs. 5(a)–5(c) correspond to our experimental results for the normalized IC function, monitored in the receiver plane by the fast detector, as the source GSM beams are transmitted through the lab-generated turbulence. The spatial coherence widths of the beams in Fig. 5 are the same as those in Fig. 3. The solid curves in Figs. 5(a)–5(c) correspond to the product of two terms: $g(x,0,z)$, exhibited in Fig. 3 and $1+B_I(x,0,z)$, displayed in Fig. 4. Comparing the rectangles and solid curves, we find that our experimental results agree reasonably well with the theoretical prediction, Eq. (6). Therefore, turbulence acts as a linear factor that enhances IC of the partially coherent source. As a result, the maximum of $|g_t(\boldsymbol{r}_1, \boldsymbol{r}_2, z)|$ can be greater than 2.0, e.g., for $\delta_c = 0.67$ mm, the maximum of $|g_t(\boldsymbol{r}_1, \boldsymbol{r}_2, z)|$ is 2.32, and for $\delta_c = 1.03$ mm, $|g_t(\boldsymbol{r}_1, \boldsymbol{r}_2, z)|$ reaches 2.40, implying non-Gaussian statistics of the field. Our experimental results are consistent with the theoretical prediction by Eq. (6).

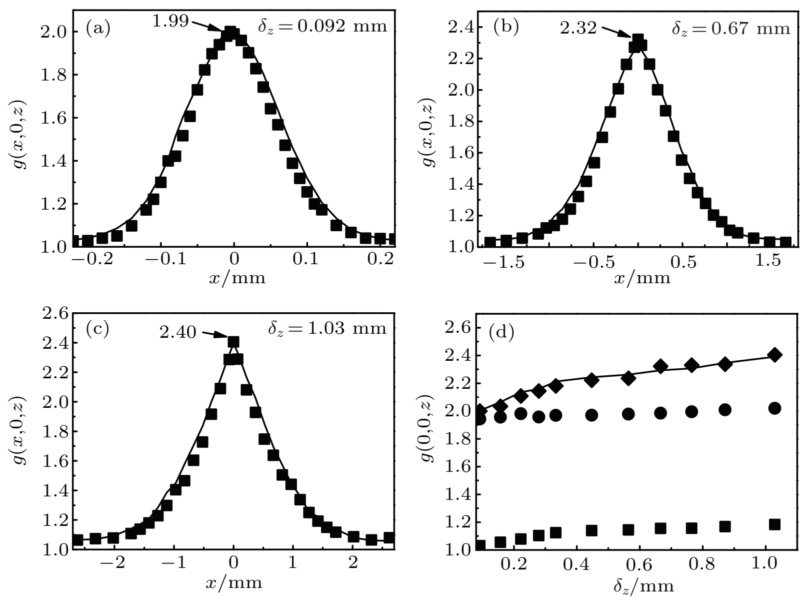


**Fig. 5.** (a)–(c) Experimental results (rectangles) for the normalized IC function of the GSM beams of different spatial coherence widths propagating through the lab-simulated turbulence as measured by the fast detector. The solid curves correspond to the product of the two terms: $g(x,0,z)$, displayed in Fig. 3 and $1+B_I(\boldsymbol{r},0,z)$, shown in Fig. 4. (d) The maximum of the normalized IC, attained at $\boldsymbol{r}_1 = \boldsymbol{r}_2 = 0$, as a function of the spatial coherence width $\delta_c$ of the GSM beam, measured by the slow detector in the turbulent medium (rectangles), fast detector in the absence of turbulence (circles), and the fast detector in the turbulent medium (diamonds). The solid curve corresponds to the product of the experimental data displayed with rectangles and circles.

Finally, we examine the dependence of the normalized IC function on the source spatial coherence. In Fig. 5(d), we exhibit the normalized IC maximum – attained at $\boldsymbol{r}_1 = \boldsymbol{r}_2 = 0$ – as determined from the measurements by the slow detector in the turbulent medium (rectangles), fast detector without turbulence (circles), and fast detector in the turbulent medium (diamonds) as functions of the source coherence width. The solid curve is obtained by multiplying the values represented with rectangles and circles. We find that the magnitude of the normalized IC maximum, determined from the fast detector measurements in the turbulent medium, increases with the spatial coherence width. This is because partially coherent beams of higher spatial coherence are more susceptible to the medium fluctuations (c.f., Fig. 4) than their low coherence counterparts. We notice, however, that as the spatial coherence width of the GSM source drops below a certain magnitude, $\delta_c < 0.1$ mm in our case, the turbulence-induced scintillations are effectively negligible, $1 + B_I \simeq 1$. Under these circumstances, effectively turbulence-free intensity correlation imaging can be realized, similar to that reported in Ref. [19]. Note that we repeated the IC measurements five times to find the experimental accuracy. It follows from our data that the relative error is about 0.5% for the free-space measurements (Fig. 3), and around 2% for the turbulent medium measurements (Figs. 4 and 5).

## 4. Discussion and conclusion

In general, intensity fluctuations in partially coherent beams can be acted as a freedom to carry information. Perhaps, one of the most remarkable applications is the ghost imaging system in which the object information is encoded into the IC functions.[22–27] When the ghost imaging system is applied in a turbulent environment, it is inevitable to involve the interaction between the intensity fluctuations of partially coherent beams and the intensity fluctuations caused by the turbulence. Accurate prediction of the IC of the partially coherent beam in atmospheric turbulence is necessary. Some previous studies revealed that the ghost imaging is immune to atmospheric turbulence,[19,30] but some showed that the turbulence has significant effects on the quality and visibility of the ghost image.[10,28,29] Our major contribution in this research is the derivation of a concise general relation between the normalized IC of partially coherent beams and the turbulence induced intensity correlations under weak fluctuation conditions tracked by a fast detector (i.e., a Hanbury Brown and Twiss type setup). We have found that turbulence acts as an additional noise source that enhances the intensity–intensity correlation function maximum. We have also measured the IC enhancement of a partially coherent beam in the turbulent medium with the fast detector. Our experimental results indicate that for a Gaussian Schell-model source, the normalized IC maximum in the lab-generated turbulence can attain the magnitude of 2.4 for the spatial coherence width $\delta_c = 1.03$ mm, clearly indicating non-Gaussian statistics of light transmitted through turbulence. However, for the GSM source of the spatial coherence width $\delta_c < 0.1$ mm, the normalized IC function is virtually independent of the medium fluctuations, making the system conducive to turbulence-free correlation imaging. We should emphasize that the validity of our main results is within the weak fluctuation region ($\sigma_R^2 < 1$), and our theoretical results show a good agreement with the experimental results when $\sigma_R^2 < 0.25$. Our findings could predict the severity of turbulence effects on IC of partially coherent light and provide instructions for the applications of ghost imaging in complex environment.

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